Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.
No calculators.

1. The coordinates of three vertices of a parallelogram are $A(1,1)$, $B(2,4)$, and $C(-5,1)$. Compute the area of the parallelogram.

2. In a circle, chord AB has length 5 and chord AC has length 7. Arc AC is twice the length of arc AB, and both arcs have degree less than 180. Compute the area of the circle.

3. Spencer eats ice cream in a right circular cone with an opening of radius 5 and a height of 10. If Spencer's ice cream scoops are always perfectly spherical, compute the radius of the largest scoop he can get such that at least half of the scoop is contained within the cone.

4. Let ABC be a triangle such that $AB = 3$, $BC = 4$, and $AC = 5$. Let X be a point in the triangle. Compute the minimal possible value of $AX^2 + BX^2 + CX^2$.

5. Let ABC be a triangle where $\angle BAC = 30^\circ$. Construct D in $\triangle ABC$ such that $\angle ABD = \angle ACD = 30^\circ$. Let the circumcircle of $\triangle ABD$ intersect AC at X. Let the circumcircle of $\triangle ACD$ intersect AB at Y. Given that $DB - DC = 10$ and $BC = 20$, find $AX \cdot AY$.

6. Let E be an ellipse with major axis length 4 and minor axis length 2. Inscribe an equilateral triangle ABC in E such that A lies on the minor axis and BC is parallel to the major axis. Compute the area of $\triangle ABC$.

7. Let ABC be a triangle with $AB = 13$, $BC = 14$, and $AC = 15$. Let D and E be the feet of the altitudes from A and B, respectively. Find the circumference of the circumcircle of $\triangle CDE$.

8. O is a circle with radius 1. A and B are fixed points on the circle such that $AB = \sqrt{2}$. Let C be any point on the circle, and let M and N be the midpoints of AC and BC, respectively. As C travels around circle O, find the area of the locus of points on MN.

9. In cyclic quadrilateral $ABCD$, $AB \cong AD$. If $AC = 6$ and $\frac{AB}{BD} = \frac{3}{5}$, find the maximum possible area of $ABCD$.

10. Let ABC be a triangle with $AB = 12$, $BC = 5$, $AC = 13$. Let D and E be the feet of the internal and external angle bisectors from B, respectively. (The external angle bisector from B bisects the angle between BC and the extension of AB.) Let ω be the circumcircle of $\triangle BDE$; extend AB so that it intersects ω again at F. Extend FC to meet ω again at X, and extend AX to meet ω again at G. Find FG.