

# Mathematical Expectations for the Stanford Math Tournament

## 1. INTRODUCTION AND CAVEATS

If you are trying to decide whether to take the General Test or the Subject Tests, or trying to decide between Subject Tests, this is a rough guideline to our expectations about the mathematical background of students taking each test. Some of the prerequisites listed below apply to almost every problem in a given test, and others apply to at most one or two of the hardest ones. Some contain assumptions about high school class curricula which your classes may violate. **To be confident in your choice, there is no substitute for looking through the previous years' tests and trying out some of the problems.**

Broadly speaking, taking a test for which you do not have the necessary background or practice is a very discouraging experience. We want each SMT participant to take a test that is appropriate. **Students with little or no contest experience may find the Subject Tests to be too difficult** and should strongly consider taking the General Test instead, while **students with significant contest experience may find the General Test too easy** and should strongly consider taking the Subject Tests instead.

## 2. SUMMARY OF TEST FORMATS

The **Power Round** is a 80-minute, multi-part, proof-oriented test. It focuses on a single topic that will be revealed and introduced at the beginning of the test.

The **Team Round** is a 50-minute exam consisting of 15 short answer questions, testing material from all areas of high school mathematics.

The **Subject Area Tests** are 50-minute individual exams consisting of 10 short answer questions. The subjects offered are Discrete, Algebra, Calculus, and Geometry. The Discrete test covers probability, number theory, and combinatorics. Students may choose any two of the subjects.

The **General Test** is a 110-minute test that is designed for students with less mathematical background. It is held at the same time as the subject area tests and contains 25 short answer questions.

## 3. TEST-BY-TEST PREREQUISITES

**3.1. General.** We strongly recommend that **students with little or no math contest experience should take the General Test.** For example, we recommend the General Test for students who have not qualified for the AIME. On the other hand, **we strongly recommend the Subject Tests for students who are comfortable with AIME-level problems.**

The test will assume comfort with basic algebra, counting and probability, geometry, and number facts. Topics include but are not limited to:

- Solving linear equations in two variables
- Solving quadratic equations
- The Pythagorean Theorem
- Similarity and congruence of geometric figures
- Area and volume formulas
- Prime and composite numbers

All problems will be solvable without trigonometry or advanced methods in algebra, combinatorics, or geometry. Knowledge of complex numbers is not necessary.

**3.2. Algebra.** The test will assume comfort with the material of a typical high school Algebra II class. Topics include but are not limited to:

- Solving quadratic equations
- Logarithms and exponentiation
- Polynomial factorizations, e.g. difference of squares, the Binomial Theorem
- Polynomial facts, e.g. solving, Vieta's formulas
- Topics in complex numbers—manipulation, conjugation, roots of unity
- Sequences and series
- Inequalities, e.g. AM-GM, Cauchy-Schwarz

3.3. **Geometry.** The test will assume comfort with the material of a typical high school Geometry class. Topics include but are not limited to:

- Properties of isosceles, equilateral, and right triangles, and of common quadrilaterals (squares, rectangles, parallelograms, etc.)
- Similarity and congruence
- Angle bisectors, altitudes, and medians in a triangle
- Triangle centers, incircles, and circumcircles
- Polygon area computations, Heron's and Brahmagupta's Formulas
- Circle facts, e.g. Power of a Point, properties of cyclic quadrilaterals
- Coordinate geometry
- 3-d geometry, including common polyhedra (tetrahedra and other pyramids, prisms, octahedra)
- Trigonometry, including the Laws of Sines and Cosines

3.4. **Discrete.** The test will assume comfort with topics in discrete mathematics, including but not limited to:

- Combinations and permutations, Principle of Inclusion-Exclusion
- Probability and expected value
- Topics in number theory such as prime factorization, base changes, modular arithmetic, the Chinese Remainder Theorem, and Fermat's Little Theorem
- Recurrences, sequences, and series

3.5. **Calculus.** The test will assume comfort with the material of the AP Calculus BC curriculum. Topics include but are not limited to:

- Limits, including L'Hôpital's Rule
- Differentiation, including the product rule and chain rule
- Integration, including  $u$ -substitution, trigonometric substitution, integration by parts
- Infinite sequences and series; Taylor series
- Basic differential equation techniques; separation of variables

Approximately the first half of the calculus test will be solvable using only concepts covered in AP Calculus AB. We strongly recommend that **students currently taking AP Calculus AB should not take the Calculus test.**

3.6. **Power and Team.** For the **Power Round**, everything listed under the subject tests is fair game.

For the **Team Round**, everything listed under the subject tests is fair game, and no one subject will dominate.