

## COMMON MATHEMATICAL NOTIONS

1. Answers should be exact and simplified. For more information on valid answer formats, see the Acceptable Answer Formats document.
2. On the Power Round, the word *compute* only calls for an answer; no explanation or proof is needed. Unless otherwise stated, all other questions require explanation or proof.
3. If a diagram is given with a problem, it is not necessarily drawn to scale.
4. In a triangle  $ABC$ , the vertices are called  $A$ ,  $B$ , and  $C$ . The sides are called  $a$ ,  $b$ , and  $c$ , with side  $a$  opposite vertex  $A$ , side  $b$  opposite vertex  $B$ , and side  $c$  opposite vertex  $C$ . If a polygon is called  $ABCDEF$ , its vertices will occur in that order around the polygon. This convention holds for all namings of polygons.
5. Unless otherwise noted, polygons (including triangles) are simple and non-degenerate.
6. If *complex numbers* are used in a problem,  $i$  denotes  $\sqrt{-1}$ .
7. The *real part* and the *imaginary part* of a complex number  $z$  are denoted by  $\operatorname{Re} z$  and  $\operatorname{Im} z$  respectively. If  $z = a + bi$  where  $a$  and  $b$  are real, then  $\operatorname{Re} z = a$  and  $\operatorname{Im} z = b$ .
8. *Logs* are base  $e$  unless otherwise indicated. When logs are used in a different base, a subscript will be used, as in  $\log_{10} 2$ . Base  $e$  logs may also be written as  $\ln 2$ .
9. The word *prime* refers to positive numbers only. Note that 1 is not a prime.
10. *Divisors* and *factors* of an integer refer to positive numbers only. *Proper divisors* of an integer refer to divisors that are less than that integer.
11. A *lattice point* is a point such that all of its coordinates are integers.
12. If a problem refers to the *digits* of a number, those digits are underlined to distinguish the digits of a number from the product of the digits. For example,  $\underline{3} \underline{1} \underline{A} \underline{B}$  refers to a four digit number and not the product  $3 \cdot 1 \cdot A \cdot B$ .
13. *Combinations* will be denoted by  $\binom{n}{k}$ ; this is the number of ways to choose  $k$  things from  $n$  things.
14. The expressions  $\arcsin x$ ,  $\sin^{-1} x$ ,  $\arccos x$ ,  $\cos^{-1} x$ ,  $\arctan x$ ,  $\tan^{-1} x$  refer to the principal values of these inverse trigonometric functions. This means that  $-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$ ,  $0 \leq \cos^{-1} x \leq \pi$ , and  $-\frac{\pi}{2} \leq \tan^{-1} x \leq \frac{\pi}{2}$ .
15. If a trigonometric problem does not specify the use of degrees, all trigonometric expressions are given in radians.
16. The *floor function* (or *greatest integer function*) is denoted by  $\lfloor x \rfloor$ , and it is defined as  $\lfloor x \rfloor = n$  when  $n \leq x < n + 1$ . Similarly, the *ceiling function* (or *least integer function*) is denoted by  $\lceil x \rceil$ , and it is defined as  $\lceil x \rceil = n$  when  $n - 1 < x \leq n$ .
17. The *fractional part* is denoted by  $\{x\}$ , and it is defined as  $\{x\} = x - \lfloor x \rfloor$ .
18. *Intervals* are written as a pair of numbers. Round brackets indicate that the endpoint is excluded, while square brackets indicate that the endpoint is included. For example, the interval  $(2, 3]$  denotes  $\{x : 2 < x \leq 3\}$ .
19. The *greatest lower bound* of a set is the largest number that is less than or equal to every number of the set. For example, the greatest lower bound of the intervals  $(2, 3)$  and  $[2, 3]$  are both 2. The *least upper bound* of a set is the largest number that is greater than or equal to every number of the set. For example, the least upper bound of intervals  $(2, 3)$  and  $[2, 3]$  are both 3.
20.  $\max\{a_1, a_2, \dots, a_n\}$  denotes the largest element in a set, and  $\min\{a_1, a_2, \dots, a_n\}$  denotes the smallest element in a set.