MATH MINGLE: WEEK 5

1. Suppose you have a bunch of unit length line segments in $\mathbb{R}^3$, and you can connect them to each other at right angles. For which integers n can you find a loop of these connected segments that is closed, i.e. the n^{th} segment connects back to the 1^{st} segment at a right angle?
2. Evaluate

   (i)
   $$\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

   (ii)
   $$\frac{1}{1} - \frac{1}{2} + \frac{1}{4} - \frac{1}{5} + \frac{1}{7} - \frac{1}{8} + \dots$$

   (iii)
   $$\frac{1}{1} - \frac{1}{2} - \frac{1}{3} + \frac{1}{4} + \frac{1}{6} - \frac{1}{7} - \frac{1}{8} + \frac{1}{9} + \dots$$

   Hint: think about constructing a pentagon with compass and straightedge.
3. From Pascal's triangle remove the "first two diagonals" and the "last two diagonals." (The first diagonal consists of all ones, as does the last diagonal.) Sum the reciprocals of all the numbers that remain: what does this equal?