MATH MINGLE: WEEK 1

1. Three fishermen sell fish at a market. One has 10 fish, another has 16, and the third has 26. Each fisherman sells at least one, but not all, of his fish before noon, each fisherman selling at the same price per fish. Later in the day, as the fish start to go bad, each sells his remaining fish at the same reduced price per fish. If each fisherman received a total of \$35.00 from the sale of his fish, what was the price before noon and the selling price after noon?

2. A quadrilateral $ABCD$ is called *balanced* if for each point P in the interior of the quadrilateral, we have
$$\text{Area}(\triangle PAB) + \text{Area}(\triangle PCD) = \text{Area}(\triangle PBC) + \text{Area}(\triangle PDA).$$
Find all balanced quadrilaterals.

3. A frog jumps in the plane with the first jump from the origin. The frog jumps distance 1 on the first jump, distance 2 on the second jump, distance 4 on the third jump and in general jumps distance 2^{n-1} on the nth jump. For each jump, the frog jumps in a straight line but can jump in any direction. Can the frog, by a judicious choice of directions, end up back at the origin on its 2012th jump?
   What is the answer if the frog jumps distance 1.5^{n-1} on its nth jump?

4. Evaluate
$$\int_0^1 \left(\left(1 - x^{2011}\right)^{1/2011} - x \right)^2 \, dx.$$

5. A warden is in charge of 100 prisoners, each of which possesses one white glove and one black glove. The warden decides to play the following game: on each prisoner's head he places a hat on which is written an arbitrary distinct real number. The prisoners may see the numbers written on all hats except their own. Then, simultaneously, each prisoner chooses to put one of their gloves on their left hand and one on their right. The warden then lines the prisoners up side-by-side with the numbers on their hats in increasing order, and the prisoners link hands (in the natural way — no arm contortions). If every pair of linked hands is wearing gloves of the same color, the prisoners go free; otherwise they are executed. The prisoners may strategize beforehand but no further communication occurs once everyone has seen the hats. What should the prisoners do?