Abstract

Every calculus textbook says that $\int e^{-x^2} \, dx$ cannot be "computed in elementary terms". But the textbooks never indicate how one can prove such a statement, or even give a precise definition of an "elementary" function, without which we cannot make a proof!

We will give a reasonable definition of elementarity, state Liouville’s necessary criterion for an integral of an elementary function to be elementary, and explain the non-trivial application of this criterion to prove that $\int e^{-x^2} \, dx$ really is not elementary. At the end we’ll give some indications about differential Galois theory, which puts these matters into a broader context.