Abstract

Ramanujan conjectured that for the sequence \( \{c_0, c_1, \ldots\} \) defined by the 2-step linear recurrence \( c_n = c_{n-1} - 2c_{n-2} \) with initial values \( c_0 = 0 \) and \( c_1 = 1 \), the only odd \( n \) for which \( c_n = -1 \) are \( n = 3, 5, 13 \). In general, what can be said about the positions where a fixed value \( k \) can occur in a linear recurrence?

The answer involves allowing \( n \) to take values in a new number system, the \( p \)-adic numbers. We will explore how to calculate in this exotic world, and apply it to other concrete problems with ordinary integers (as well as to understand one reason why Euler was justified in proposing that the divergent sum \( 1 + 2 + 4 + 8 + \ldots \) should be assigned the value \(-1\)).