

Stanford University Mathematical Organization (SUMO) Speaker Series

Why is the Classification of Finite Simple Groups not a zoo?

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3:00–4:00 PM @ 380-380Y

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Abstract

One of the greatest accomplishments of 20th century mathematics was the determination of all finite simple groups: the cyclic groups of prime size, the alternating groups A_n for $n \geq 5$, 26 exceptional cases (called “sporadic”), and a huge list of “matrix groups over finite fields”. In effect, nearly all examples arise from linear algebra over finite fields in a precise sense.

A primary reason for the value of a classification is using it to prove a general result by checking all cases, but navigating an enormous list of different types of matrix groups over different finite fields may seem likely to be a nightmare in practice. But this is not so: there is a *uniform* way to think about the zoo of such groups, and that perspective often makes it possible to establish results for all of them by systematic arguments avoiding case-by-case tedium.

We will give a glimpse into this unified viewpoint, assuming familiarity with groups at the level of Math 120 and with finite fields.