Consider the following two sets of fractions:

\[ A = \left\{ 0, \frac{1}{5}, \frac{1}{3}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{4}{5} \right\}, \quad B = \left\{ 0, \frac{1}{30}, \frac{7}{30}, \frac{11}{30}, \frac{13}{30}, \frac{17}{30}, \frac{19}{30}, \frac{23}{30}, \frac{29}{30} \right\}. \]

Thus, \( A \) contains all the reduced fractions in \( [0, 1) \) with denominators 1, 2, 3, 5 and \( B \) contains all the reduced fractions in \( [0, 1) \) with denominator 30. Amazingly, the fractions in \( A \) and \( B \) interlace: if you arrange each list in ascending order, you’ll see that a fraction in \( A \) is followed by one in \( B \) and vice-versa. This is related to an observation of Chebyshev that

\[
\frac{(30n)!n!}{(15n)!(10n)!(6n)!}
\]

is an integer for all natural numbers \( n \). I will explain work towards classifying such sets of interlacing fractions, and the related question of finding integral factorial ratios.