Stanford University Mathematical Organization (SUMO) Speaker Series

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6-7pm, Arrillaga Study Room

Fractions!

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Abstract

Consider the following two sets of fractions:

$$A = \left\{0, \frac{1}{5}, \frac{1}{3}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{4}{5}\right\}, \quad B = \left\{0, \frac{1}{30}, \frac{7}{30}, \frac{11}{30}, \frac{13}{30}, \frac{17}{30}, \frac{19}{30}, \frac{23}{30}, \frac{29}{30}\right\}.$$

Thus, A contains all the reduced fractions in $[0, 1)$ with denominators $1, 2, 3, 5$ and B contains all the reduced fractions in $[0, 1)$ with denominator 30. Amazingly, the fractions in A and B interlace: if you arrange each list in ascending order, you'll see that a fraction in A is followed by one in B and vice-versa. This is related to an observation of Chebyshev that

$$\frac{(30n)!n!}{(15n)!(10n)!(6n)!}$$

is an integer for all natural numbers n. I will explain work towards classifying such sets of interlacing fractions, and the related question of finding integral factorial ratios.