Fractions!

Prof. Kannan Soundararajan

Abstract

Consider the following two sets of fractions:

\[ A = \left\{ 0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5} \right\} \quad \text{and} \quad B = \left\{ \frac{7}{30}, \frac{11}{30}, \frac{13}{30}, \frac{19}{30}, \frac{23}{30}, \frac{29}{30} \right\} \]

Thus, \( A \) contains all the reduced fractions in \([0, 1)\) with denominators 1, 2, 3, 5, and \( B \) contains all the reduced fractions in \([0, 1)\) with denominator 30. Amazingly, the fractions in \( A \) and \( B \) interlace: if you arrange all the fractions in ascending order, you will see that a fraction in \( A \) will be followed by one in \( B \) followed by one in \( A \) and so on! This is also related to an observation of Chebyshev that

\[ \frac{(30n)!n!}{(15n)!(10n)!(6n)!} \]

is an integer for all natural numbers \( n \). I will explain work towards classifying all such sets of interlacing fractions, and the related question of finding integral factorial ratios.