Stanford University Mathematical Organization (SUMO) Speaker Series

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6-7pm, Ricker Dining Hall

Rational points on conics

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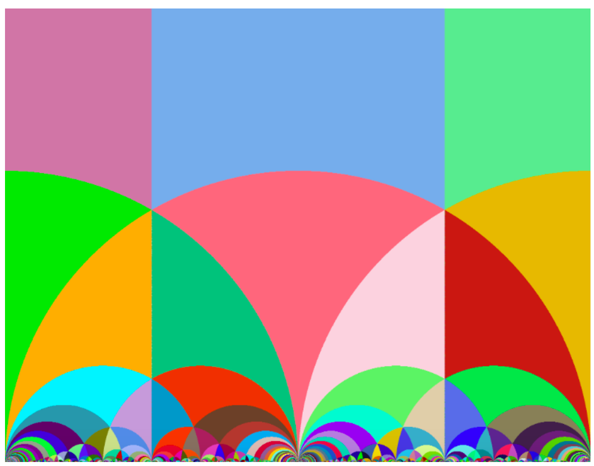

Abstract

Describing all Pythagorean triples (i.e., integer solutions to the Pythagorean equation) reduces to describing all points on $u^2+v^2=1$ with rational coordinates, which can be done via a geometric method. The technique describes the rational points on any conic $au^2+bv^2=c$ (with rational a, b, c) *provided* there is at least one rational point (e.g., $(-1,0)$ for the unit circle). But how can we determine if a given conic has a rational point at all?

For instance, consider the conics $7u^2-15v^2=23$ and $7u^2-15v^2=-23$. The first has no rational points (why not?) and the second has a rational point (where?). How can one systematically analyze this question? We will discuss Gauss' solution, which spawned many notions that were fundamental for the subsequent development of number theory.