1. Compute
\[ \int_{0}^{10} (x - 5) + (x - 5)^2 + (x - 5)^3 \, dx. \]

**Answer:** \( \frac{250}{3} \)

**Solution:** This integral is equivalent to
\[ \int_{-5}^{5} x + x^2 + x^3 \, dx = \int_{-5}^{5} x^2 \, dx = \frac{5^3}{3} - \frac{(-5)^3}{3} = \frac{250}{3}. \]

2. Water is flowing out through the smaller base of a hollow conical frustum formed by taking a downwards pointing cone of radius 12m and slicing off the tip of the cone in a cut parallel to the base so that the radius of the cross-section of the slice is 6m (meaning the smaller base has a radius of 6m). The height of the frustum is 10m. If the height of the water level in the frustum is decreasing at 3m/s and the current height is 5m, then the volume of the water in the frustum is decreasing at \( \frac{d}{dt} m^3/s. \) Compute \( d. \)

**Answer:** 243

**Solution:** Let the radii of the bases of the frustum be \( r_1 = 6 \) and \( r_2 = 12, \) and let the height be \( h = 10. \) Also, let the current height of the water in the frustum be \( h_c = 5, \) the radius of the surface of the water be \( r_c, \) and the height of the part of the cone that was cut off to make the frustum be \( h_0. \)

First, we can find \( h_0 \) using similar triangles. We have
\[ \frac{h_0}{h_0 + h} = \frac{r_1}{r_2} \Rightarrow h_0 r_2 = h_0 r_1 + h r_1 \Rightarrow h_0 = \frac{h r_1}{r_2 - r_1} = \frac{10 \cdot 6}{12 - 6} = 10. \]

We can also find \( r_c \) in terms of \( h_c: \)
\[ \frac{r_c}{r_1} = \frac{h_0 + h_c}{h_0} \Rightarrow r_c = r_1 \frac{h_0 + h_c}{h_0}. \]

The volume of the water in the frustum is
\[ V = \frac{1}{3} \pi r_c^2 (h_0 + h_c) - \frac{1}{3} \pi r_1^2 h_0 = \frac{1}{3} \pi r_1^2 \frac{(h_0 + h_c)^2}{h_0^2} (h_0 + h_c) - \frac{1}{3} \pi r_1^2 h_0. \]

Taking the derivative with respect to time gives
\[ \frac{dV}{dt} = \frac{1}{3} \pi r_1^2 \cdot \frac{3 (h_0 + h_c)^2}{h_0^2} \cdot \frac{dh_c}{dt} = \frac{1}{3} \pi \cdot 6^2 \cdot 3 \left( \frac{(10 + 5)^2}{10^2} \right) \cdot 3 = \frac{243 \pi}{10}. \]

3. Compute the value of
\[ \int_{-\pi}^{\pi} e^{x^2} - e^{-x^2} |x| \, dx. \]

**Answer:** \( \frac{1}{2} \ln \left( e^{2\pi^2} - 2\pi^2 \right) \)
**Solution:** Let $I$ denote the value of the integral. We first split the integral into two parts

$$I = \int_{-\pi}^{0} \frac{e^{x^2} - e^{-x^2}}{e^{x^2} - x\sqrt{2}} \cdot |x| \, dx + \int_{0}^{\pi} \frac{e^{x^2} - e^{-x^2}}{e^{x^2} - x\sqrt{2}} \cdot |x| \, dx.$$

Since $|x| = x$ for $x > 0$ and $|x| = -x$ for $x < 0$, we have

$$I = \int_{-\pi}^{0} \frac{e^{x^2} - e^{-x^2}}{e^{x^2} - x\sqrt{2}}(-x) \, dx + \int_{0}^{\pi} \frac{e^{x^2} - e^{-x^2}}{e^{x^2} - x\sqrt{2}} \, x \, dx.$$

Performing a substitution $x \rightarrow -x$ on the first integral gives

$$I = \int_{0}^{\pi} \frac{e^{x^2} - e^{-x^2}}{e^{x^2} + x\sqrt{2}} \, x \, dx + \int_{0}^{\pi} \frac{e^{x^2} - e^{-x^2}}{e^{x^2} - x\sqrt{2}} \, x \, dx = \int_{0}^{\pi} \frac{(2e^{2x^2} - 2)x}{e^{2x^2} - 2x^2} \, dx.$$

Now, the indefinite integral can be solved by performing the substitution $v = e^{2x^2} - 2x^2$.

$$\int \frac{(2e^{2x^2} - 2)x}{e^{2x^2} - 2x^2} \, dx = \int \frac{dv}{2v} = \frac{1}{2} \ln v.$$

Therefore, $\int_{0}^{\pi} \frac{(2e^{2x^2} - 2)x}{e^{2x^2} - 2x^2} \, dx = \left[ \frac{1}{2} \ln v \right]_{0}^{\pi} = \left[ \frac{1}{2} \ln \left( e^{2x^2} - 2x^2 \right) \right]_{0}^{\pi} = \frac{1}{2} \ln \left( e^{2\pi^2} - 2\pi^2 \right)$. 