

Time limit: 50 minutes.

Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.

No calculators.

1. Let $ABCD$ be a unit square. A semicircle with diameter AB is drawn so that it lies outside of the square. If E is the midpoint of arc AB of the semicircle, what is the area of triangle CDE ?
2. A cat and mouse live on a house mapped out by the points $(-1, 0)$, $(-1, 2)$, $(0, 3)$, $(1, 2)$, $(1, 0)$. The cat starts at the top of the house (point $(0, 3)$) and the mouse starts at the origin $(0, 0)$. Both start running clockwise around the house at the same time. If the cat runs at 12 units a minute and the mouse at 9 units a minute, how many laps around the house will the cat run before it catches the mouse?
3. In triangle ABC with $AB = 10$, let D be a point on side BC such that AD bisects $\angle BAC$. If $\frac{CD}{BD} = 2$ and the area of ABC is 50, compute the value of $\angle BAD$ in degrees.
4. Let ω_1 and ω_2 be two circles intersecting at points P and Q . The tangent line closer to Q touches ω_1 and ω_2 at M and N respectively. If $PQ = 3$, $QN = 2$, and $MN = PN$, what is QM^2 ?
5. The bases of a right hexagonal prism are regular hexagons of side length $s > 0$, and the prism has height h . The prism contains some water, and when it is placed on a flat surface with a hexagonal face on the bottom, the water has depth $\frac{s\sqrt{3}}{4}$. The water depth doesn't change when the prism is turned so that a rectangular face is on the bottom. Compute $\frac{h}{s}$.
6. Let the altitude of $\triangle ABC$ from A intersect the circumcircle of $\triangle ABC$ at D . Let E be a point on line AD such that $E \neq A$ and $AD = DE$. If $AB = 13$, $BC = 14$, and $AC = 15$, what is the area of quadrilateral $BDCE$?
7. Let G be the centroid of triangle ABC with $AB = 9$, $BC = 10$, and $AC = 17$. Denote D as the midpoint of BC . A line through G parallel to BC intersects AB at M and AC at N . If BG intersects CM at E and CG intersects BN at F , compute the area of triangle DEF .
8. In the coordinate plane, a point A is chosen on the line $y = \frac{3}{2}x$ in the first quadrant. Two perpendicular lines l_1 and l_2 intersect at A where l_1 has slope $m > 1$. Let l_1 intersect the x -axis at B , and l_2 intersects the x and y axes at C and D , respectively. Suppose that line BD has slope $-m$ and $BD = 2$. Compute the length of CD .
9. Let $ABCD$ be a quadrilateral with $\angle ABC = \angle CDA = 45^\circ$, $AB = 7$, and $BD = 25$. If AC is perpendicular to CD , compute the length of BC .
10. Let ABC be an acute triangle with $BC = 48$. Let M be the midpoint of BC , and let D and E be the feet of the altitudes drawn from B and C to AC and AB respectively. Let P be the intersection between the line through A parallel to BC and line DE . If $AP = 10$, compute the length of PM .