1. How many ways are there to choose positive integers $x$ and $y$ such that the lowest common multiple of $x$ and $y$ is 216?
   
   **Answer:** 49
   
   **Solution:** Note that $216 = 2^33^3$. First, consider the power of 2. $x$ must be divisible by 1 of $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, and $2^3 = 8$; similarly, $y$ must be divisible by 1 of these 4 powers of 2. However, we cannot have both $x$ and $y$ be divisible by only $2^0$, $2^1$, or $2^2$ and not $2^3$. Hence, there are $4 \cdot 4 - 3 \cdot 3 = 7$ ways of distributing the powers of 2. Similarly, there are 7 ways of distributing the powers of 3. Hence, the number of ways to choose positive integers $x$ and $y$ is $7 \cdot 7 = 49$.

2. Consider tangent circles $\gamma_1$ and $\gamma_2$ with centers $O_1, O_2$ and radii $R, r$ with $r < R$, respectively. Let $\overline{AB}$ be a common external tangent of length 16. The area of $\triangle ABO_1O_2$ is 160. Find the ordered pair $(r, R)$.
   
   **Answer:** $(4, 16)$
   
   **Solution:** By the area of a trapezoid, $160 = \frac{16 \times (r + R)}{2}$. So, $r + R = 20$. Also, Pythagorean Theorem gives us $(R - r)^2 + 16^2 = (R + r)^2$. So, $R - r = 12$. Solving gives the answer.

3. Consider the set of odd integers $S = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21\}$. Let $\wp(S)$ denote the set of subsets of $S$. Given $T \in \wp(S)$, we define to $\alpha_T$ to be the sum of the elements of $T$. Compute $\sum_{T \in \wp(S)} \alpha_T$.
   
   **Answer:** 123904
   
   **Solution:** Each number in $S$ appears in the same number of subsets: namely, $2^{10} = 1024$ of them. Then the final answer is 1024 times the sum of the elements of $S$. Conveniently, the sum of the first $n$ odd numbers is $n^2$ so the sum of $S$ is $11^2 = 121$. The final answer is then $121 \times 1024 = 123904$. Pro tip: it’s easiest to do this computation by multiplying 1024 by 11 twice, since multiplying by 11 is pretty easy.

More generally, if we replace $S$ with the set of the first $n$ odd numbers, the answer will be $2^{n-1}n^2$. 