**Time limit:** 50 minutes.
**Instructions:** This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
**No calculators.**

1. David owns a parking lot for vehicles. A vehicle is either a motorcycle with two wheels or a car with four wheels. Today, there are 100 vehicles parked in his parking lot. The total number of wheels in David's parking lot is 326. If David collects \$1.00 from each motorcycle and \$2.00 from each car per day, how much money in dollars does David collect today?

2. Three consecutive terms of a geometric sequence of positive integers multiply to $1,000,000$. If the common ratio is greater than 1, what is the smallest possible sum of the three terms?

3. Let $x, y$ be real numbers such that
$$x + y = 2,$$
$$x^4 + y^4 = 1234.$$
Find $xy$.

4. Evaluate $(350 + 90\sqrt{15})^{\frac{1}{3}} + (350 - 90\sqrt{15})^{\frac{1}{3}}$.

5. Let $f(x) = 36x^4 - 36x^3 - x^2 + 9x - 2$. Then let the four roots of $f(x)$ be $r_1$, $r_2$, $r_3$, and $r_4$. Find the value of
$$(r_1 + r_2 + r_3)(r_1 + r_2 + r_4)(r_1 + r_3 + r_4)(r_2 + r_3 + r_4).$$

6. Let $f(X)$ be a complex monic quadratic with real roots $\frac{1}{3}, \frac{2}{3}$. (The polynomial $f(X)$ is of the form $X^2 + bX + c$ where $b, c, X$ are complex numbers.) If $|z| = 1$, what is the sum of all possible values of $f(z)$ such that $f(z) = \overline{f(z)}$?

7. Given that $x, y$ are real numbers satisfying $x > y > 0$, compute the minimum value of
$$\frac{5x^2 - 2xy + y^2}{x^2 - y^2}.$$

8. The equation
$$(x-1)(x-2)(x-4)(x-5)(x-7)(x-8) = (x-3)(x-6)(x-9)$$
has distinct roots $r_1, r_2, \ldots, r_6$. Evaluate
$$\sum_{i=1}^{6} (r_i - 1)(r_i - 2)(r_i - 4).$$

9. Suppose we have a strictly increasing function $f : \mathbb{Z}^+ \to \mathbb{Z}^+$ where $\mathbb{Z}^+$ denotes the set of positive integers. We also know that both
$$f(f(1)), f(f(2)), f(f(3)), \ldots$$

and

$$f(f(1)+1), f(f(2)+1), f(f(3)+1), \ldots$$

are arithmetic sequences. Given that $f(1) = 1$ and $f(2) = 3$, find the maximum value of

$$\sum_{j=1}^{100} f(j).$$

10. Let $\mathbb{R}_{\geq 0}$ be the set of nonnegative real numbers. Consider a continuous function $f : \mathbb{R}_{\geq 0} \to \mathbb{R}$ which satisfies

$$f(x^2) + f(y^2) = f\left(\frac{x^2y^2 - 2xy + 1}{x^2 + 2xy + y^2}\right)$$

for $x, y$ positive real numbers with $xy > 1$. Given that $f(0) = 2019$ and $f(1) = \frac{2019}{2}$, compute $f(3)$.