

**Time limit:** 50 minutes.

**Instructions:** This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.

**No calculators.**

1. Consider a semi-circle with diameter  $AB$ . Let points  $C$  and  $D$  be on diameter  $AB$  such that  $CD$  forms the base of a square inscribed in the semicircle. Given that  $CD = 2$ , compute the length of  $AB$ .
2. Let  $ABCD$  be a trapezoid with  $AB$  parallel to  $CD$  and perpendicular to  $BC$ . Let  $M$  be a point on  $BC$  such that  $\angle AMB = \angle DMC$ . If  $AB = 3$ ,  $BC = 24$ , and  $CD = 4$ , what is the value of  $AM + MD$ ?
3. Let  $ABC$  be a triangle and  $D$  be a point such that  $A$  and  $D$  are on opposite sides of  $BC$ . Give that  $\angle ACD = 75^\circ$ ,  $AC = 2$ ,  $BD = \sqrt{6}$ , and  $AD$  is an angle bisector of both  $\triangle ABC$  and  $\triangle BCD$ , find the area of quadrilateral  $ABDC$ .
4. Let  $a_1, a_2, \dots, a_{12}$  be the vertices of a regular dodecagon  $D_1$  (12-gon). The four vertices  $a_1, a_4, a_7, a_{10}$  form a square, as do the four vertices  $a_2, a_5, a_8, a_{11}$  and  $a_3, a_6, a_9, a_{12}$ . Let  $D_2$  be the polygon formed by the intersection of these three squares. If we let  $[A]$  denotes the area of polygon  $A$ , compute  $\frac{[D_2]}{[D_1]}$ .
5. In  $\triangle ABC$ ,  $\angle ABC = 75^\circ$  and  $\angle BAC$  is obtuse. Points  $D$  and  $E$  are on  $AC$  and  $BC$ , respectively, such that  $\frac{AB}{BC} = \frac{DE}{EC}$  and  $\angle DEC = \angle EDC$ . Compute  $\angle DEC$  in degrees.
6. In  $\triangle ABC$ ,  $AB = 3$ ,  $AC = 6$ , and  $D$  is drawn on  $BC$  such that  $AD$  is the angle bisector of  $\angle BAC$ .  $D$  is reflected across  $AB$  to a point  $E$ , and suppose that  $AC$  and  $BE$  are parallel. Compute  $CE$ .
7. Two equilateral triangles  $ABC$  and  $DEF$ , each with side length 1, are drawn in 2 parallel planes such that when one plane is projected onto the other, the vertices of the triangles form a regular hexagon  $AFBDCE$ . Line segments  $AE$ ,  $AF$ ,  $BF$ ,  $BD$ ,  $CD$ , and  $CE$  are drawn, and suppose that each of these segments also has length 1. Compute the volume of the resulting solid that is formed.
8. Let  $ABC$  be a right triangle with  $\angle ACB = 90^\circ$ ,  $BC = 16$ , and  $AC = 12$ . Let the angle bisectors of  $\angle BAC$  and  $\angle ABC$  intersect  $BC$  and  $AC$  at  $D$  and  $E$  respectively. Let  $AD$  and  $BE$  intersect at  $I$ , and let the circle centered at  $I$  passing through  $C$  intersect  $AB$  at  $P$  and  $Q$  such that  $AQ < AP$ . Compute the area of quadrilateral  $DPQE$ .
9. Let  $ABCD$  be a cyclic quadrilateral with  $3AB = 2AD$  and  $BC = CD$ . The diagonals  $AC$  and  $BD$  intersect at point  $X$ . Let  $E$  be a point on  $AD$  such that  $DE = AB$  and  $Y$  be the point of intersection of lines  $AC$  and  $BE$ . If the area of triangle  $ABY$  is 5, then what is the area of quadrilateral  $DEYX$ ?
10. Let  $ABC$  be a triangle with  $AB = 13$ ,  $AC = 14$ , and  $BC = 15$ , and let  $\Gamma$  be its incircle with incenter  $I$ . Let  $D$  and  $E$  be the points of tangency between  $\Gamma$  and  $BC$  and  $AC$  respectively, and let  $\omega$  be the circle inscribed in  $CDIE$ . If  $Q$  is the intersection point between  $\Gamma$  and  $\omega$  and  $P$  is the intersection point between  $CQ$  and  $\omega$ , compute the length of  $PQ$ .