

1. Eric writes problems at a constant rate of 6 problems per hour, and Elizabeth solves problems at a constant rate of 10 problems per hour. If Eric starts writing problems 2 hours before Elizabeth starts solving problems, how many problems will he have written when Elizabeth finishes solving all the problems he has written so far?

Answer: 30

Solution: After two hours, Eric will have written 12 problems. Let x be the number of hours that Elizabeth solves problems. Eric will have written $12 + 6x$ problems, while Elizabeth will have solved $10x$ problems. This gives us the equation $12 + 6x = 10x$, which solves to $x = 3$. Therefore, Elizabeth will have solved $10 \cdot 3 = \boxed{30}$ problems.

2. Lake Donald is invaded by 1000 ducks with red or blue feathers and red or blue heads. If 538 of the ducks have red feathers, 318 ducks have blue heads and 250 ducks have both blue heads and blue feathers. How many ducks have both red heads and red feathers?

Answer: 470

Solution: Because 318 ducks have blue heads and 250 ducks have both blue heads and blue feathers, there are $318 - 250 = 68$ ducks with red heads and blue feathers. The 538 ducks with red feathers include ducks with both red and blue heads, so if we remove the 68 ducks with both red heads and blue feathers, we are left with $538 - 68 = \boxed{470}$ ducks with both red heads and red feathers.

3. Maddy has a chocolate coin in the shape of a cylinder with radius 3 and height $\frac{1}{2}$. Maddy also has a gumball in the shape of a sphere with the same volume as her chocolate coin. What is the radius of her gumball?

Answer: $\frac{3}{2}$

Solution: Recall that the volume of a cylinder with height h and radius r is $V = \pi r^2 h$. Likewise, the volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$. Therefore, the volume of the chocolate coin is $\pi \cdot 3^2 \cdot \frac{1}{2} = \frac{9\pi}{2}$. If Maddy's gumball has radius r , we get the equation

$$\frac{9\pi}{2} = \frac{4}{3}\pi r^3. \text{ Solving for } r \text{ gives us the answer } r = \boxed{\frac{3}{2}}.$$

4. Katy only owns two types of books: comic books and nature books. $\frac{1}{3}$ of her books are comic books. After going to a booksale, she buys 20 more comic books, so $\frac{4}{7}$ of her books are now comic books. How many books did she have originally?

Answer: 36

Solution: Let x be the number of comic books that Katy started with and let y be the number of nature books that Katy started with. This gives us the system of equations

$$\frac{x}{x+y} = \frac{1}{3} \qquad \text{and} \qquad \frac{x+20}{x+y+20} = \frac{4}{7}.$$

Simplifying the two equations gives us $2x = y$ and $3x + 60 = 4y$. Solving the system of equations gives us $x = 12$ and $y = 24$, so Katy started with $\boxed{36}$ books.

5. A line passes through $(-2, 1)$ and $(4, 4)$. Point $(7, y)$ is also on this line. Compute y .

Answer: $\frac{11}{2}$

Solution: Observe that the distance between $(4, 4)$ and $(7, y)$ is half the distance between $(4, 4)$ and $(-2, 1)$. This gives us the equation $y - 4 = \frac{1}{2}(4 - 1)$, which solves to $y = \boxed{\frac{11}{2}}$.

6. Eric has a 9-sided dice and Harrison has an 11-sided dice. They each roll their respective die. Eric wins if he rolls a number greater or equal to Harrison's number. What is the probability Eric wins?

Answer: $\frac{5}{11}$

Solution: There are $9 \cdot 11 = 99$ possible combinations of rolls. We then count the number of ways Eric can win:

- If Eric rolls a 1, Harrison must roll a 1.
- If Eric rolls a 2, Harrison must roll a 1 or 2.
- In general, if Eric rolls an n , then Harrison must roll any number between 1 and n , for a total of n possible rolls.

Thus, the number of combinations where Eric wins is $1 + 2 + \dots + 9 = 45$. The probability

Eric wins is therefore $\frac{45}{99} = \frac{5}{11}$.

7. A number is formed using the digits $\{2, 0, 1, 8\}$, using all 4 digits exactly once. Note that $0218 = 218$ is a valid number that can be formed. What is the probability that the resulting number is strictly greater than 2018?

Answer: $\frac{11}{24}$

Solution: There are $4! = 24$ total possible permutations of the digits $\{2, 0, 1, 8\}$. Next, we count the number of cases where the permutation is greater than 2018. When the leftmost digit is an 8, the permutation must be greater than 2018, and there are $3! = 6$ such permutations. When the leftmost digit is a 2, the permutation is not greater than 2018 only when the permutation is 2018 itself, since $0 < 1 < 8$. In this case, there are $3! - 1 = 5$ such permutations. Because all other permutations of the digits have a thousands digit which is less than 2, the desired probability is $\frac{6+5}{24} = \frac{11}{24}$.

8. A regular hexagon with side length 1 has a circle inscribed in it. Then another regular hexagon is inscribed in the circle. What is the area of the inner hexagon?

Answer: $\frac{9\sqrt{3}}{8}$

Solution: Notice that the diameter of the inscribed circle is the height of the outer hexagon. If we decompose the inner and outer hexagons into 6 equilateral triangles, we see that the side length of one of the inner triangles is equal to the height of one of the outer triangles. Because the side length of an outer equilateral triangle is 1, its height is $\frac{\sqrt{3}}{2}$, so the side length of the inner hexagon is also $\frac{\sqrt{3}}{2}$. It is known that the area of an equilateral triangle with side length s is $\frac{\sqrt{3}}{4}s^2$, so the area of the inner hexagon is therefore $6 \cdot \frac{\sqrt{3}}{4} \cdot \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{9\sqrt{3}}{8}$.

9. Ed writes the first 2018 positive integers down in order: $1, 2, 3, \dots, 2018$. Then for each power of 2 that appears, he crosses out that number as well as the number 1 greater than that power of 2. After he is done, how many numbers are not crossed out?

Answer: 1997

Solution: There are 11 powers of 2 less than 2018: $2^0, 2^1, \dots, 2^{10} = 1024$. For each power of 2, we cross out two numbers. However, we end up crossing out 2 twice, because it is crossed out by $2^0 + 1$ and by 2^1 . Therefore, the total number of integers we cross out is $11 \cdot 2 - 1 = 21$. It follows that $2018 - 21 = \boxed{1997}$ numbers are left.

10. The sum of the digits of Anna's age is Carly's age. In 10 years, Anna will be twice as old as Carly. What is the sum of their ages?

Answer: 34

Solution: We begin by assuming that Anna's age is a 2-digit number. A proof for why this must be true is below.

Let us represent Anna's age as \underline{AB} , and let us represent Carly's age as \underline{C} .

The first sentence in the problem gives us the equation $A + B = C$, while the second sentence gives us the equation $10A + B + 10 = 2(C + 10)$. Eliminating C from both equations yields $8A = 10 + B$. Now B must be between 0 and 9 inclusive, so the RHS must be between 10 and 19 inclusive. However, there is only one multiple of 8 in that range, so the only solution is $A = 2$ and $B = 6$. Therefore, Anna is 26 and Carly is 8, giving us the answer $26 + 8 = \boxed{34}$.

For completeness, we now show that Anna's age must be a 2-digit number. Let x be Anna's age and y be Carly's age.

Suppose that x has n digits. The smallest n -digit number is 10^{n-1} , so $10^{n-1} \leq x$. Furthermore, the largest sum of digits for an n -digit number occurs when all n digits are 9's, giving us $y \leq 9n$. Now in 10 years, Anna will be twice as old as Carly, so $x + 10 = 2(y + 10)$. Plugging in our inequalities, we get

$$10^{n-1} + 10 \leq x + 10 = 2(y + 10) \leq 18n + 20.$$

Therefore, n must satisfy

$$10^{n-1} \leq 18n + 10.$$

Because 10^{n-1} grows much faster than $18n$, we can easily see that this inequality is only satisfied when $n \leq 2$. However, if $n = 1$, then we get $x = y$, which implies $x + 10 = 2(x + 10)$. Solving yields $x = -10$, which is absurd, and therefore $n = 2$.

11. The sequence 2, 3, 5, 6, 7, 8, 10, ... contains all positive integers that are not perfect squares. Find the 2018th term of the sequence.

Answer: 2063

Solution: Observe that $44^2 = 1936$ and $45^2 = 2025$. Therefore, the sequence 2, 3, ..., 2018 has $2018 - 44 = 1974$ terms. To find the 2018th term, we need to add 44 numbers from 2018 to 2062. But those numbers will also include $45^2 = 2025$, so the 2018th term is actually $\boxed{2063}$.

12. Let $ABCDE$ be a regular pentagon. Label points F on BC and G on ED such that AFG is an equilateral triangle and $FG \parallel CD$. Compute $\angle AFB$.

Answer: 48

Solution: Using the formula for computing the sum of the interior angles of a polygon, the interior angle sum for a regular pentagon is $(5 - 2) \cdot 180^\circ = 540^\circ$. Therefore, each interior angle is $\frac{540^\circ}{5} = 108^\circ$. Now because $FG \parallel CD$, we have $\angle GFC + \angle FCD = 180^\circ$, so $\angle GFC = 180^\circ - 108^\circ = 72^\circ$. Next, we have $\angle AFB + \angle AFG + \angle GFC = 180^\circ$, and because $\triangle AFG$ is equilateral, we have $\angle AFG = 60^\circ$. Plugging in values for $\angle AFG$ and $\angle GFC$, we solve to get $\angle AFB = 180^\circ - 60^\circ - 72^\circ = \boxed{48^\circ}$.

13. Consider the sequence $a_1 = 2, a_2 = 3, a_3 = 6, a_4 = 18, \dots$, where $a_n = a_{n-1} \cdot a_{n-2}$. What is the largest k such that 3^k divides a_{11} ?

Answer: 55

Solution: Note that a_n contains only powers of 2 and 3 for all $n \geq 1$. This allows us to write $a_n = 2^{b_n} 3^{c_n}$ for some positive integers b_n, c_n for all $n \geq 1$. Because $a_n = a_{n-1} \cdot a_{n-2}$, we have

$2^{b_n} 3^{c_n} = 2^{b_{n-1}+b_{n-2}} 3^{c_{n-1}+c_{n-2}}$, which implies that $b_n = b_{n-1} + b_{n-2}$ and $c_n = c_{n-1} + c_{n-2}$ for all $n \geq 1$. Note that the recurrence for c_n looks exactly like the recurrence for the sequence of Fibonacci numbers, defined as $F_1 = F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$. Because $c_2 = c_3 = 1$, it is simply the sequence of Fibonacci numbers shifted forwards one term, so we compute $c_{11} = F_{10} = \boxed{55}$.

14. Harrison the astronaut is trying to navigate his way through a rectangular grid in outer space. He starts at $(0, 0)$ and needs to reach the Earth at position $(6, 6)$. Harrison can only move upwards or rightwards. Unfortunately, there are two black holes, which are unit squares, with lower left corners at $(1, 4)$ and $(3, 1)$. If Harrison steps onto any corner of a black hole, he gets sucked in and won't be able to return home. How many paths can Harrison take to get back to Earth safely?

Answer: 208

Solution: Note that to get to from (A, B) to $(A + M, B + N)$, Harrison must go right (denoted as R) M times and up (denoted as U) N times. We can then represent his path by a string containing M R s and N U s. This gives us a total of $\binom{M+N}{M}$ such paths.

Drawing the grid of space, we find that there are 3 main ways for Harrison to return to Earth safely.

- Case 1: Harrison goes above the two black holes. Harrison must then first move to $(0, 6)$, of which there is $\binom{0+6}{0} = 1$ way, and then move to $(6, 6)$, of which there is again $\binom{6+0}{6} = 1$ way. This gives us a total of $1 \cdot 1 = 1$ paths above the two black holes.
- Case 2: Harrison goes between the two black holes. Harrison must then first move to $(2, 3)$, then $(3, 3)$ and finally to $(6, 6)$. There are $\binom{2+3}{2} = 10$ ways to get from $(0, 0)$ to $(2, 3)$, 1 way to get from $(2, 3)$ to $(3, 3)$ and $\binom{3+3}{3} = 20$ ways to get from $(3, 3)$ to $(6, 6)$. This gives us a total of $10 \cdot 1 \cdot 20 = 200$ paths between the two black holes.
- Case 3: Harrison goes below the two black holes. Harrison must then first move to $(5, 0)$, of which there is $\binom{5+0}{5} = 1$ way, and then move to $(6, 6)$ of which there are $\binom{1+6}{1} = 7$ ways. This gives us a total of $1 \cdot 7 = 7$ paths below the two black holes.

Together, there are $1 + 200 + 7 = \boxed{208}$ possible paths for Harrison to get back to Earth.

15. How many ways are there to select distinct integers x, y , where $1 \leq x \leq 25$ and $1 \leq y \leq 25$, such that $x + y$ is divisible by 5?

Answer: 120

Solution: There are two cases:

- $x \equiv 0 \pmod{5}$. This implies $y \equiv 0 \pmod{5}$. There are 5 ways to choose x , and 4 ways to choose y to be distinct from x , giving us a total of $5 \cdot 4 = 20$ possibilities.
- $x \not\equiv 0 \pmod{5}$. This implies $y \equiv -x \pmod{5}$. Note that if $y \equiv x \pmod{5}$, then we would have $2x \equiv 0 \pmod{5}$, which implies $x \equiv 0 \pmod{5}$ because $\gcd(2, 5) = 1$, a contradiction. Therefore, x and y cannot be the same value in this case, so there are always exactly 5 such y satisfying this equation. There are 20 ways of choosing x , and each choice of x can match with 5 choices of y , giving us a total of $20 \cdot 5 = 100$ possibilities.

In total there are $20 + 100 = \boxed{120}$ possible choices of x and y .

16. A turtle and rabbit both stand at the corner of a cube with side length 18 meters and race to get to the opposite corner of the cube. The turtle can swim through the interior of the cube at a rate of 6 meters/minute. The rabbit can run at a rate of 9 meters/minute, but

must go along the faces of the cube. The rabbit arrives at the opposite corner first. How long must he wait for the turtle to arrive?

Answer: $3\sqrt{3} - 2\sqrt{5}$

Solution: First we calculate the time it takes the turtle to reach the opposite corner. The inner diagonal is the hypotenuse of the triangle consisting of an edge and a face diagonal. We can use the Pythagorean Theorem to first compute the length of a face diagonal, which is $\sqrt{(18)^2 + (18)^2} = 18\sqrt{2}$, and use this to compute the length of the inner diagonal, which is $\sqrt{(18)^2 + (18\sqrt{2})^2} = 18\sqrt{3}$. Because the turtle travels at a rate of 6 meters/minute, it will take $\frac{18\sqrt{3}}{6} = 3\sqrt{3}$ minutes to reach the opposite corner.

We then compute the time it takes the rabbit to reach the opposite corner. If we unwrap the cube, we see that the shortest path from one corner to the opposite corner is in fact on the hypotenuse of a triangle with side lengths 18 and 36. This path therefore has length $\sqrt{(18)^2 + (36)^2} = 18\sqrt{5}$. Because the rabbit travels at a rate of 9 meters/minute, it will take $\frac{18\sqrt{5}}{9} = 2\sqrt{5}$ minutes to reach the opposite corner.

Therefore, the rabbit must wait $\boxed{3\sqrt{3} - 2\sqrt{5}}$ minutes.

17. Two squares of side length 3 overlap so that the shared region is a square of side length 1. Compute the area of the smallest hexagon that covers the 2 squares.

Answer: 21

Solution: The area of the squares themselves is $2 \cdot 3^2 - 1^2 = 17$, since we have to consider the area of the overlapping section. Next, we observe that the desired hexagon adds 2 additional right triangles to the area, with each right triangle having a base of $3 - 1 = 2$ and a height of $3 - 1 = 2$. Therefore, the total area is $17 + 2 \cdot \frac{2 \cdot 2}{2} = \boxed{21}$.

18. How many integer pairs (a, b) satisfy $\frac{1}{a} + \frac{1}{b} = \frac{1}{2018}$?

Answer: 17

Solution: We can rewrite this equation as $2018(a + b) = ab$. Using Simon's factoring trick, we have $(a - 2018)(b - 2018) = 2018^2$. Factoring, we get $2018^2 = 2^2 \cdot 1009^2$. This gives us $(2 + 1)(2 + 1) = 9$ possible factorizations into two positive factors. Each pair of factors also has its negative counterparts, giving us a total of 18 factors. However, the factorization $2018 \cdot 2018 = 2018^2$ yields the solution $(a, b) = (0, 0)$, which is not valid in our original equation. Thus, we have $\boxed{17}$ pairs.

19. A 3×3 magic square is a grid of **distinct** numbers whose rows, columns, and diagonals all add to the same integer sum. Connie creates a magic square whose sum is N , but her keyboard is broken so that when she types a number, one of the digits (0–9) always appears as a different digit (e.g. if the digit 8 always appears as 5, the number 18 will appear as 15). The altered square is shown below. Find N .

9	11	10
18	17	6
14	11	15

Answer: 51

Solution: First, notice that 11 appears in the square twice. This is not possible in the original magic square, because the numbers must be distinct, so we conclude that the output of the broken key is 1.

Next, we see that the digits 0, 1, 4, 5, 6, 7, 8, 9 are all present in the square. Therefore, the broken key must be 2 or 3. We then calculate the sum of the rows and columns as shown:

9	11	10	30
18	17	6	41
14	11	15	40
41	39	31	

Each row has a distinct sum, so there must be at least two altered squares. The middle row is $18 + 17 + 6 = 41$. Since none of the ones digits are 1, the ones digit of N is the same as the ones digit of 41.

The ones digits in the $9 + 11 + 10 = 30$ row needs to sum up to end in 1, so the broken key must be 2. Similarly, the ones digit in $14 + 11 + 15 = 40$ needs to be 1, so both of the 11's actually end with a 2.

9	12	10
18	17	6
14	12	15

They cannot both be 12's, so one of them is a 22. This means that the sum of the middle column is either $12 + 22 + 17 = 51$ or $12 + 22 + 27 = 61$. Looking at the top row, even if all of the 1's are changed into 2's, the maximum value of the row is $9 + 22 + 20 = 51$. Therefore, the sum of each row and column is $N = \boxed{51}$.

The original square is shown below, with the modified digits in bold:

9	22	20
28	17	6
14	12	25

20. Let $ABCD$ be a convex quadrilateral with $AB = \sqrt{2}$, $CD = 2$, and $BD = 1 + \sqrt{3}$. If $\angle ABD = 45^\circ$ and $\angle BDC = 30^\circ$, what is the length of AC ?

Answer: 2

Solution: Let E be the point on BD such that $AE \perp BD$. Then $\triangle ABE$ is a 45-45-90 triangle so $BE = AE = 1$. This implies that $DE = \sqrt{3}$.

We also know that $\angle BDC = 30^\circ$, $CD = 2$, and $DE = \sqrt{3}$, so $\triangle CDE$ is a 30-60-90 triangle. Therefore, $CE = 1$ and $\angle CED = 90^\circ$.

However, $\angle AEB = 90^\circ$ and B , E , and D are collinear, so A , E , and C are also collinear. It follows that $AC = AE + CE = 1 + 1 = \boxed{2}$.

21. Positive integer n has the property such that $n - 64$ is a positive perfect cube. Suppose that n is divisible by 37. What is the smallest possible value of n ?

Answer: 407

Solution: Let k be a positive integer such that $n - 64 = k^3$. Note that minimizing n is equivalent to minimizing k . Adding 64 to both sides and factoring gives us

$$n = (k + 4)(k^2 - 4k + 16).$$

Since 37 is prime, we guess that either $k + 4 = 37$ or $k^2 - 4k + 16 = 37$. In the former case, we have $k = 33$. In the latter case, we subtract 37 from both sides and factor to get

$$(k - 7)(k + 3) = 0.$$

Since k must be positive, the only solution is $k = 7$. Therefore, the smallest possible value of n is achieved when $k = 7$, which gives us $n = k^3 + 64 = 343 + 64 = \boxed{407}$.

22. One of the six digits in the expression $435 \cdot 605$ can be changed so that the product is a perfect square N^2 . Compute N .

Answer: 495

Solution: We first factorize $435 = 3 \cdot 5 \cdot 29$ and $605 = 5 \cdot 11^2$. First, suppose that we change 605. In this case, we could change it to be of the form $(3 \cdot 5 \cdot 29)k^2$ for some positive integer k . If $k = 1$, we clearly cannot change 605 to 435 by only modifying a single digit, and if $k \geq 2$, then this number would have more than 4 digits.

Therefore, we must change 435 so that it is of the form $5k^2$. It must be divisible by 5, so the last digit must be 0 or 5. If we change the last digit to a 0, then we have $430 = 5k^2 \implies k^2 = 86$, which is not a perfect square. Therefore, the last digit must remain a 5, which also implies that k must be odd. Computing $5k^2$ for small odd values of k gives us the three digit numbers 125, 245, 405, 605, and 845. The only number that can be formed by changing a single digit of 435 is $405 = 3^4 \cdot 5$, so we have $N^2 = 405 \cdot 605 = 3^4 \cdot 5^2 \cdot 11^2$. Taking the square root gives us the answer $N = 3^2 \cdot 5 \cdot 11 = \boxed{495}$.

23. James throws a dart at a dartboard, which is shaped like a regular hexagon of side length 2 feet. What is the probability that James throws the dart within 1 foot of any of the 6 corners of the hexagon?

Answer: $\frac{2\pi\sqrt{3}}{18}$

Solution: The desired probability is the ratio between the circular areas at each of the 6 corners of the hexagon and the area of the hexagon itself.

The area of an equilateral triangle of side length 2 is $2^2 \cdot \frac{\sqrt{3}}{4} = \sqrt{3}$, so the area of a regular hexagon of side length 2 is $6\sqrt{3}$.

On the other hand, each of the circular areas at each corner of a hexagon is exactly $\frac{1}{3}$ of an entire circle. Hence, the total area of the circular areas is $6 \cdot \frac{1}{3} \cdot \pi \cdot 1^2 = 2\pi$.

Therefore, our desired probability is $\frac{2\pi}{6\sqrt{3}} = \boxed{\frac{2\pi\sqrt{3}}{18}}$.

24. Stu is on a train en route to SMT. He is bored, so he starts doodling in his notebook. Stu realizes that that he can combine *SMT* as an alphametic, where each letter represents a unique integer and the leading digits may not be zero, to get his name as shown: $\sqrt{SMT} + SMT = STU$. Find the three digit number *STU*.

Answer: 650

Solution: The numbers 10 through 31 have 3-digit perfect squares, so $10 \leq \sqrt{SMT} \leq 31$. Each letter represents a distinct number, so there are no repeated digits in *SMT*. The hundreds digit does not change when we add \sqrt{SMT} to *SMT* to form *STU*, so the tens digit of *STU* must be greater than the tens digit of *SMT*, which in turn implies that $M < T$. The only squares that fulfill these conditions are

$$13^2 = 169, 16^2 = 256, 17^2 = 289, 18^2 = 324, 23^2 = 529, 25^2 = 625, 27^2 = 729$$

Now, we can try computing $\sqrt{SMT} + SMT$ for each of the squares to check if the sum is of

the form STU :

$$13 + 169 = 182 \neq 19?$$

$$16 + 256 = 272 \neq 26?$$

$$17 + 289 = 306 \neq 29?$$

$$18 + 324 = 342 = 34?$$

$$23 + 529 = 552 \neq 59?$$

$$25 + 625 = 650 = 65?$$

$$27 + 729 = 756 \neq 79?$$

Both $SMT = 324$ and $SMT = 625$ appear to work. However, if $SMT = 324$, then we end up with $M = U = 2$ which is disallowed. On the other hand, if $SMT = 625$ then $STU = 650$ gives us $U = 0$, which has not yet been used. Therefore, the only solution is $STU = \boxed{650}$.

25. Let $f(x) = x^3 - n_1x^2 + (n_2 - k^2)x - (n_3 - k^4)$. Suppose that n_1, n_2 , and n_3 form a geometric sequence with common ratio k and that the roots of f are nonzero and form an arithmetic sequence with common difference also k . Find k .

Answer: 3

Solution: Let the roots of f be r_1, r_2 , and r_3 , where $r_1 \leq r_2 \leq r_3$. Because we know the common difference of this sequence is k , we have $r_1 = r_2 - k$ and $r_3 = r_2 + k$.

From Vieta's formulas, we have

$$\begin{aligned} n_1 &= (r_2 - k) + (r_2) + (r_2 + k) \\ n_2 - k^2 &= (r_2)(r_2 - k) + (r_2)(r_2 + k) + (r_2 - k)(r_2 + k) \\ n_3 - k^4 &= (r_2)(r_2 - k)(r_2 + k) \end{aligned}$$

Simplifying these equations gives us

$$\begin{aligned} n_1 &= 3r_2 \\ n_2 &= 3r_2^2 \\ n_3 &= r_2^3 - r_2k^2 + k^4 \end{aligned}$$

Because n_1, n_2, n_3 form a geometric series with common ratio k , we have $kn_1 = n_2$ and $k^2n_1 = n_3$. Plugging in the values above into the first equation, we have $3kr_2 = 3kr_2^2$, which gives us $k = r_2$. Therefore, $n_3 = k^3 - k^3 + k^4 = k^4$. Plugging these values into $k^2n_1 = n_3$ gives us $k^2 \cdot (3k) = k^4$, which solves to give us $k = \boxed{3}$.