1. Positive integer n has 6 factors including n and 1. Suppose that the 3rd largest factor of n, including n, is 55. Compute n.

   Answer: 605

   Solution: Since n has 6 factors, n must be written in the form ab^2 for some primes a, b, as this is the only way to express a number with $(1+1)(2+1) = 2 \cdot 3 = 6$ factors. Because 55 is a factor of n, we must have either $a = 5, b = 11$ or $a = 11, b = 5$. Note that in the second case, $n = 11 \cdot 5^2 = 275$, which would result in 55 being the 2nd largest factor of n. Hence, we must have $a = 5, b = 11$, resulting in $x = 5 \cdot 11^2 = \boxed{605}$.

2. How many 5 digit numbers n exist such that each n is divisible by 9 and *none* of the digits of n are divisible by 9?

   Answer: 3640

   Solution: For each number of digits (e.g. 1-digit numbers, 2-digit numbers, and so on), we consider how many numbers n do not have 0 or 9 as a digit and are divisible by 9, and how many numbers n do not have 0 or 9 as a digit and are not divisible by 9.

   - For 1-digit numbers n, there are exactly 8 numbers without a 0 or a 9, none of which are divisible by 9. Hence, there are 0 1-digit numbers divisible by 9 and 8 1-digit numbers not divisible by 9.
   - For 2-digit numbers n, any n divisible by 9 can be formed by taking a 1-digit number m not divisible by 9 and adding the unique digit that will form a number divisible by 9. The remaining numbers will not be divisible by 9. Hence, there are 8 2-digit numbers divisible by 9 and $7 \cdot 8 + 0 = 56$ 2-digit numbers not divisible by 9.
   - For 3-digit numbers n, we can take any of the 56 2-digit numbers not divisible by 9 and add a digit to form a 3-digit number divisible by 9. The the remaining numbers will not be divisible by 9. Hence, there are 56 3-digit numbers divisible by 9, and $7 \cdot 56 + 8 \cdot 8 = 456$ 3-digit numbers not divisible by 9.
   - For 4-digit numbers n, we can take any of the 456 3-digit numbers not divisible by 9 and add a digit to form a 4-digit number divisible by 9. The remaining numbers will not be divisible by 9. Hence, there are 456 4-digit numbers divisible by 9, and $7 \cdot 456 + 8 \cdot 56 = 3640$ 4-digit numbers not divisible by 9.
   - For 5-digit numbers n, we can take any of the 3640 4-digit numbers not divisible by 9 to form a 5-digit number divisible by 9. The remaining numbers will not be divisible by 9. Hence, there are 3640 5-digit numbers are divisible by 9.

   We conclude that there are $\boxed{3640}$ numbers which satisfy the problem's conditions.

3. A string t is *regular* if it consists of n '(' characters followed by n ')' characters where $n \geq 1$ (i.e. '()', "(())", "((()))", etc.). Define $f(s)$ to be the number of ways that we can remove characters from a string s to form a regular string. For example, $f(\text{"(())"}) = 5$, since we can either remove no characters to obtain the regular string "(())", or we can remove 2 characters 4 different ways to obtain the regular string "()". Compute the sum of $f(s)$ over all strings s of length 10 formed by using '(' and ')' characters.

   Answer: 28501

   Solution: Rather than computing $f(s)$ for each string s individually, we instead count how many times each regular string appears in a string of length 10 formed by using '(' and ')' characters.

   For a regular string t of length $2n$ where $1 \leq n \leq 5$, we first choose which of the ten positions will contain t's characters. This can be done in $\binom{10}{2n}$ ways. The remaining characters can be assigned in 2^{10-2n} ways, so t is counted $\binom{10}{2n}2^{10-2n}$ times over all possible strings. Summing over all n, we find that the answer is

$$\binom{10}{2}2^8 + \binom{10}{4}2^6 + \binom{10}{6}2^4 + \binom{10}{8}2^2 + \binom{10}{10}2^0 = \boxed{28501}.$$