1. If \( f(x) = \frac{g(x)}{x} \) where \( g(2) = 4 \) and \( g'(2) = 8 \), compute \( f'(2) \).

**Answer: 3**

**Solution:** We use the quotient rule to compute
\[
 f'(x) = \frac{g'(x) \cdot x - g(x) \cdot 1}{x^2}
\]
\[
 f'(2) = \frac{8 \cdot 2 - 4 \cdot 1}{2^2} = \frac{16 - 4}{4} = 3
\]

2. Compute
\[
 \lim_{n \to 1} \sum_{k=1}^{25} \frac{n^k - 1}{\ln(n)}.
\]

**Answer: 325**

**Solution:** We begin by applying L’Hopital’s Rule to each term in the sum
\[
 \lim_{n \to 1} \frac{n^k - 1}{\ln(n)} = \lim_{n \to 1} \frac{k n^{k-1}}{1/n} = k.
\]
If we swap the limit and finite sum in our original problem, we can then compute
\[
 \lim_{n \to 1} \sum_{k=1}^{25} \frac{n^k - 1}{\ln(n)} = \sum_{k=1}^{25} \lim_{n \to 1} \frac{n^k - 1}{\ln(n)} = \sum_{k=1}^{25} k = \frac{(25)(26)}{2} = 325.
\]

3. Compute
\[
 \int_{0}^{\pi} \sin^{10}(x)\,dx.
\]

**Answer: \( \frac{63\pi}{256} \)**

**Solution:** We first compute
\[
 \int_{0}^{\pi} \sin^2(x)\,dx = \int_{0}^{\pi} \frac{1 - \cos(2x)}{2}\,dx
\]
\[
 = \left( \frac{x}{2} - \frac{\sin(2x)}{4} \right) \bigg|_{0}^{\pi} = \frac{\pi}{2}.
\]

Now note that \( d(-\cos(x)) = \sin(x) \), which allows us to integrate by parts and compute
\[
 \int_{0}^{\pi} \sin^n(x)\,dx = \int_{0}^{\pi} \sin^{n-1}(x)d(-\cos(x))\,dx
\]
\[
 = -\sin^{n-1}(x)\cos(x) \bigg|_{0}^{\pi} - \int_{0}^{\pi} (n-1) \sin^{n-2}(-\cos^2(x))\,dx
\]
\[
 = 0 + (n-1) \int_{0}^{\pi} \sin^{n-2}(1 - \sin^2(x))\,dx
\]
\[
 = (n-1) \int_{0}^{\pi} \sin^{n-2}(x)\,dx - (n-1) \int_{0}^{\pi} \sin^n(x)\,dx.
\]
Rearranging gives us
\[
 \int_{0}^{\pi} \sin^n(x)\,dx = \frac{n-1}{n} \int_{0}^{\pi} \sin^{n-2}(x)\,dx.
\]
This gives us a recursive relation between the integrals. From above, when $n = 2$, we have
\[ \int_0^\pi \sin^2(x) \, dx = \frac{\pi}{2}. \]
Therefore, when $n = 10$, we have
\[
\frac{\pi}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{7}{8} \cdot \frac{9}{10} = \frac{63\pi}{256}.
\]