1. There is a circular table with 6 chairs. Alice, Bob, and Eve each pick a random chair to sit down in. Compute the probability that each person is sitting next to two empty chairs.

**Answer:** $\frac{1}{10}$

**Solution:** If Alice picks a seat at random to sit down in, Bob then has probability $\frac{2}{5}$ of sitting down in a valid seat (since if he sits directly across from Alice, Eve will be forced to sit next to one of them), and Eve then has probability $\frac{1}{4}$ of sitting down in a valid seat, for a probability of $\frac{1}{10}$.

2. $ABCDEF$ is a regular hexagon, and diagonal $AC$ has length 6. Find the area of hexagon $ABCDEF$.

**Answer:** $18\sqrt{3}$

**Solution:** Let $M$ be the midpoint of $AC$. Then $ABM$ is a $30-60-90$ right triangle. Since $AM = 3$, $AB = 2\sqrt{3}$, so the area of an equilateral triangle with side length $AB$ is $(2\sqrt{3})^2 \cdot \frac{\sqrt{3}}{4} = 3\sqrt{3}$. Hexagon $ABCDEF$ is composed of six such triangles, for a total area of $18\sqrt{3}$.

3. Let $n$ be the number which consists of the first 2014 positive integers concatenated together. Let $f(x)$ be the sum of the digits of $x$, and let $g(x)$ be the value obtained by applying $f$ repeatedly to $x$ until it converges to a single value. Compute $g(n)$.

**Answer:** 1

**Solution:** Two integers are equivalent mod some integer $k$ if they have the same remainder upon division by $k$. Note that $g(x)$ is equivalent to $f(x)$ (mod 9). Furthermore, $f(x)$ should converge to a single-digit number. Since the sum of the digits $1 - 9$ is 0 (mod 9), the sum of the digits of the numbers $1 - 999$ is also 0 (mod 9) and the sum of the digits of the numbers $1000 - 1999$ is $1000 \equiv 1$ (mod 9). Finally, we can calculate that the sum of the digits of the numbers $2000 - 2014$ is 0 (mod 9). Thus, the sum of the digits of $n$ is 1 (mod 9), and since a number mod 9 is equivalent to the sum of its digits mod 9, it follows that $g(x) = 1$. 