1. Compute $\int_{2012}^{2014} \left[(x-2013)^2 + (x-2015)^2\right] dx$.

   Answer: $\mathbf{\frac{28}{3}}$

   Solution:

   $\int_{2012}^{2014} (x-2013)^2\, dx = \int_{-1}^{1} x^2\, dx = \frac{2}{3}$. $\int_{2012}^{2014} (x-2015)^2\, dx = \int_{1}^{3} x^2\, dx = \frac{26}{3}$. $\frac{2}{3} + \frac{26}{3} = \boxed{\frac{28}{3}}$.

2. Let $f(x) = (x^2+8x+20)e^x$. Find $f^{(100)}(x)$.

   Answer: $\mathbf{(x^2 + 208x + 10720)e^x}$

   Consider any function of the form $g(x) = h(x)e^x$. We claim that

$$g^{(n)}(x) = e^x \sum_{k=0}^{n} \binom{n}{k} h^{(k)}(x).$$

   Given this formula, we can compute

$$f^{(100)}(x) = e^x \left((x^2+8x+20) + 100(2x+8) + \frac{100 \times 99}{2} \times 2 \right) = \boxed{(x^2 + 208x + 10720)e^x}.$$

   To prove the formula, we induct on n. The base case $n = 0$ clearly holds. Now, assume

$$g^{(n)}(x) = e^x \sum_{k=0}^{n} \binom{n}{k} h^{(k)}(x).$$

   The derivative, by the product rule is

$$\begin{aligned}
g^{(n+1)}(x) &= g^{(n)}(x) + e^x \sum_{k=0}^{n} \binom{n}{k} h^{(k+1)}(x) \\
&= g^{(n)}(x) + e^x \sum_{k=1}^{n+1} \binom{n}{k-1} h^{(k)}(x) \\
&= e^x \left(h(x) + h^{(n+1)}(x) + \sum_{k=1}^{n} \left(\binom{n}{k} + \binom{n}{k-1} \right) h^{(k)}(x) \right) \\
&= e^x \left(h(x) + h^{(n+1)}(x) + \sum_{k=1}^{n} \binom{n+1}{k} h^{(k)}(x) \right) \\
&= e^x \sum_{k=0}^{n+1} \binom{n}{k} h^{(k)}(x).
\end{aligned}$$

   Thus, the proof is complete.

3. Calculate the infinite series

$$\sum_{k=1}^{\infty} \frac{1}{2k^2 - k}.$$

   Answer: $\mathbf{2 \ln 2}$

Solution: We have the partial fraction decomposition

$$\frac{1}{2k^2 - k} = \frac{2}{2k-1} - \frac{2}{2k}.$$

Therefore,

$$\sum_{k=1}^{\infty} \frac{1}{2k^2 - k} = 2\sum_{k=1}^{\infty} \left(\frac{1}{2k-1} - \frac{1}{2k} \right) = 2\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots \right) = \boxed{2\ln 2}.$$