

**Time limit:** 15 minutes.

**Instructions:** This tiebreaker contains 3 short answer questions. All answers must be expressed in simplest form unless specified otherwise. You will submit answers to the problem as you solve them, and may solve problems in any order. You will be informed if an answer submission is correct or incorrect upon submission. Resubmissions are allowed, but incorrect submissions incur a penalty if the question is ultimately solved correctly. In addition, to prevent excessive guessing, after making an incorrect submission, you may not make another submission for 30 seconds.

**No calculators.**

1. In  $\triangle ABC$ , the altitude to  $\overline{AB}$  from  $C$  partitions  $\triangle ABC$  into two smaller triangles, each of which is similar to  $\triangle ABC$ . If the side lengths of  $\triangle ABC$  and of both smaller triangles are all integers, find the smallest possible value of  $AB$ .
2. Four points  $O$ ,  $A$ ,  $B$ , and  $C$  satisfy  $OA = OB = OC = 1$ ,  $\angle AOB = 60^\circ$ , and  $\angle BOC = 90^\circ$ .  $B$  is between  $A$  and  $C$  (i.e.  $\angle AOC$  is obtuse). Draw three circles  $O_a$ ,  $O_b$ , and  $O_c$  with diameters  $OA$ ,  $OB$ , and  $OC$ , respectively. Find the area of region inside  $O_b$  but outside  $O_a$  and  $O_c$ .
3. Circles with centers  $O_1$ ,  $O_2$ , and  $O_3$  are externally tangent to each other and have radii  $1$ ,  $\frac{1}{2}$ , and  $\frac{1}{4}$ , respectively. Now for  $i > 3$ , let circle  $O_i$  be defined as the circle externally tangent to circles  $O_{i-1}$  and  $O_{i-2}$  with radius  $2^{1-i}$  that is farther from  $O_{i-3}$ . As  $n$  approaches infinity, the area of triangle  $O_1O_2O_n$  approaches the value  $A$ . Find  $A$ .