1. A circle with radius 1 has diameter $AB$. $C$ lies on this circle such that $\widehat{AC} / \widehat{BC} = 4$. $\overline{AC}$ divides the circle into two parts, and we will label the smaller part Region I. Similarly, $\overline{BC}$ also divides the circle into two parts, and we will denote the smaller one as Region II. Find the positive difference between the areas of Regions I and II.

2. In trapezoid $ABCD$, $BC \parallel AD$, $AB = 13$, $BC = 15$, $CD = 14$, and $DA = 30$. Find the area of $ABCD$.

3. Let $ABC$ be an equilateral triangle with side length 1. Draw three circles $O_a$, $O_b$, and $O_c$ with diameters $\overline{BC}$, $\overline{CA}$, and $\overline{AB}$, respectively. Let $S_a$ denote the area of the region inside $O_a$ and outside of $O_b$ and $O_c$. Define $S_b$ and $S_c$ similarly, and let $S$ be the area of the region inside all three circles. Find $S_a + S_b + S_c - S$.

4. Let $ABCD$ be a rectangle with area 2012. There exist points $E$ on $AB$ and $F$ on $CD$ such that $DE = EF = FB$. Diagonal $AC$ intersects $DE$ at $X$ and $EF$ at $Y$. Compute the area of triangle $EXY$.

5. What is the radius of the largest sphere that fits inside an octahedron of side length 1?

6. A red unit cube $ABCDEFGH$ (with $E$ below $A$, $F$ below $B$, etc.) is pushed into the corner of a room with vertex $E$ not visible, so that faces $ABFE$ and $ADHE$ are adjacent to the wall and face $EFGH$ is adjacent to the floor. A string of length 2 is dipped in black paint, and one of its endpoints is attached to vertex $A$. How much surface area on the three visible faces of the cube can be painted black by sweeping the string over it?

7. Let $ABC$ be a triangle with incircle $O$ and side lengths 5, 8, and 9. Consider the other tangent line to $O$ parallel to $BC$, which intersects $AB$ at $B_a$ and $AC$ at $C_a$. Let $r_a$ be the inradius of triangle $AB_aC_a$, and define $r_b$ and $r_c$ similarly. Find $r_a + r_b + r_c$.

8. Let $ABC$ be a triangle with side lengths 5, 6, and 7. Choose a radius $r$ and three points outside the triangle $O_a$, $O_b$, and $O_c$, and draw three circles with radius $r$ centered at these three points. If circles $O_a$ and $O_b$ intersect at $C$, $O_b$ and $O_c$ intersect at $A$, $O_c$ and $O_a$ intersect at $B$, and all three circles intersect at a fourth point, find $r$.

9. In quadrilateral $ABCD$, $m \angle ABD = m \angle BCD$ and $\angle ADB = \angle ABD + \angle BDC$. If $AB = 8$ and $AD = 5$, find $BC$.

10. A large flat plate of glass is suspended $\sqrt{2/3}$ units above a large flat plate of wood. (The glass is infinitely thin and causes no funny refractive effects.) A point source of light is suspended $\sqrt{6}$ units above the glass plate. An object rests on the glass plate of the following description. Its base is an isosceles trapezoid $ABCD$ with $AB || DC$, $AB = AD = BC = 1$, and $DC = 2$. The point source of light is directly above the midpoint of $CD$. The object’s upper face is a triangle $EFG$ with $EF = 2$, $EG = FG = \sqrt{3}$. $G$ and $AB$ lie on opposite sides of the rectangle $EFCD$. The other sides of the object are $EA = ED = 1$, $FB = FC = 1$, and $GD = GC = 2$. Compute the area of the shadow that the object casts on the wood plate.