Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.
No calculators.

1. What is $\int_0^{10} (x-5)+(x-5)^2+(x-5)^3\,dx$?

2. Find the maximum value of
$$\int_{-\pi/2}^{3\pi/2} \sin(x) f(x)\,dx$$
subject to the constraint $|f(x)| \leq 5$.

3. Calculate
$$\int_{2^5}^{3^5} \frac{1}{x - x^{3/5}}\,dx.$$

4. Compute the x-coordinate of the point on the curve $y=\sqrt{x}$ that is closest to the point $(2,1)$.

5. Let
$$f(x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5},$$
and set $g(x) = f^{-1}(x)$. Compute $g^{(3)}(0)$.

6. Compute
$$\lim_{x\to 0} \left(\frac{\sin x}{x}\right)^{\frac{1}{1-\cos x}}.$$

7. A differentiable function g satisfies
$$\int_0^x (x-t+1)g(t)\,dt = x^4 + x^2$$
for all $x \geq 0$. Find $g(x)$.

8. Compute
$$\int_0^\infty \frac{\ln x}{x^2+4}\,dx.$$

9. Find the ordered pair (α, β) with non-infinite $\beta \neq 0$ such that $\lim\limits_{n\to\infty} \dfrac{\sqrt[n^2]{1!2!\cdots n!}}{n^\alpha} = \beta$ holds.

10. Find the maximum of
$$\int_0^1 f(x)^3\,dx$$
given the constraints
$$-1 \leq f(x) \leq 1, \quad \int_0^1 f(x)\,dx = 0.$$