1. The quadratic $x^2 - 4x + 2$ has two distinct roots, a and b. Compute the sum of the coefficients of the monic quadratic with roots $a^2 + b^2$ and $a^3 + b^3$.

   Answer: 429

   Solution: We know that $a + b = 4$ and $ab = 2$ from Vieta's. $a^2 + b^2 = (a + b)^2 - 2ab = 12$, while $a^3 + b^3 = (a + b)^3 - 3ab(a + b) = 40$. Therefore, our quadratic is $(x - 12)(x - 40)$, and the sum of the coefficients of the quadratic can be obtained by evaluating the quadratic at $x = 1$, which gives us $\boxed{429}$.

2. Find the minimum value of $xy + xz + yz$ given that x, y, z are real and $x^2 + y^2 + z^2 = 1$.

   Answer: $-\frac{1}{2}$

   Solution: We note that $(x+y+z)^2 = (x^2+y^2+z^2)+2(xy+xz+yz) = 1+2(xy+xz+yz)$. Since $(x + y + z)^2$ clearly has a minimum value of 0, it follows that $xy + xz + yz$ is at least $\boxed{-1/2}$. This occurs, for example, when $x = -y = 1/\sqrt{2}$ and $z = 0$.

3. Find all solutions α with $0° < \alpha < 90°$ to the equation $1 + \sqrt{3}\tan(60° - \alpha) = \dfrac{1}{\sin\alpha}$.

   Answer: $\mathbf{30°, 50°}$

   Solution: Multiply both sides by $\cos(60° - \alpha)$ and write 1 as $2\sin(30°)$ and $\sqrt{3}$ as $2\cos(30°)$ to get

   $$2\left(\sin(30°)\cos(60° - \alpha) + \cos(30°)\sin(60° - \alpha)\right) = \frac{\cos(60° - \alpha)}{\sin(\alpha)}.$$

   Applying the formula for the sine of a sum, we see that the left-hand-side is $2\cdot\sin(30°+60°-\alpha)$, or $2\cos(\alpha)$. Multiplying both sides by $\sin(\alpha)$ yields $2\sin(\alpha)\cos(\alpha) = \cos(60° - \alpha)$, which can be rewritten

   $$\sin(2\alpha) = \sin(\alpha + 30°). \tag{1}$$

   Therefore, either $2\alpha = \alpha + 30°$ or $2\alpha + (\alpha + 30°) = 180°$, from which we obtain $\alpha = 30°$ and $\alpha = 50°$, respectively.