Time limit: 50 minutes.

Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.

No calculators.

1. Triangle $ABC$ has side lengths $BC = 3$, $AC = 4$, $AB = 5$. Let $P$ be a point inside or on triangle $ABC$ and let the lengths of the perpendiculars from $P$ to $BC$, $AC$, $AB$ be $D_a$, $D_b$, $D_c$ respectively. Compute the minimum of $D_a + D_b + D_c$.

2. Pentagon $ABCDE$ is inscribed in a circle of radius 1. If $\angle DEA \cong \angle EAB \cong \angle ABC$, $m\angle CAD = 60^\circ$, and $BC = 2DE$, compute the area of $ABCDE$.

3. Let circle $O$ have radius 5 with diameter $\overline{AE}$. Point $F$ is outside circle $O$ such that lines $FA$ and $FE$ intersect circle $O$ at points $B$ and $D$, respectively. If $FA = 10$ and $m\angle FAE = 30^\circ$, then the perimeter of quadrilateral $ABDE$ can be expressed as $a + b\sqrt{2} + c\sqrt{3} + d\sqrt{6}$, where $a$, $b$, $c$, and $d$ are rational. Find $a + b + c + d$.

4. Let $ABC$ be any triangle, and $D, E, F$ be points on $\overline{BC}$, $\overline{CA}$, $\overline{AB}$ such that $CD = 2BD$, $AE = 2CE$ and $BF = 2AF$. $\overline{AD}$ and $\overline{BE}$ intersect at $X$, $\overline{BE}$ and $\overline{CF}$ intersect at $Y$, and $\overline{CF}$ and $\overline{AD}$ intersect at $Z$. Find $\frac{\text{Area}(\triangle XYZ)}{\text{Area}(\triangle ABC)}$.

5. Let $ABCD$ be a cyclic quadrilateral with $AB = 6$, $BC = 12$, $CD = 3$, and $DA = 6$. Let $E, F$ be the intersection of lines $AB$ and $CD$, lines $AD$ and $BC$ respectively. Find $EF$.

6. Two parallel lines $l_1$ and $l_2$ lie on a plane, distance $d$ apart. On $l_1$ there are an infinite number of points $A_1, A_2, A_3, \cdots$, in that order, with $A_nA_{n+1} = 2$ for all $n$. On $l_2$ there are an infinite number of points $B_1, B_2, B_3, \cdots$, in that order and in the same direction, satisfying $B_nB_{n+1} = 1$ for all $n$. Given that $A_1B_1$ is perpendicular to both $l_1$ and $l_2$, express the sum $\sum_{i=1}^{\infty} \angle A_iB_iA_{i+1}$ in terms of $d$.

7. In a unit square $ABCD$, find the minimum of $\sqrt{2}AP + BP + CP$ where $P$ is a point inside $ABCD$.

8. We have a unit cube $ABCDEFGH$ where $ABCD$ is the top side and $EFGH$ is the bottom side with $E$ below $A$, $F$ below $B$, and so on. Equilateral triangle $BDG$ cuts out a circle from the cube’s inscribed sphere. Find the area of the circle.

9. We have a circle $O$ with radius 10 and four smaller circles $O_1, O_2, O_3, O_4$ of radius 1 which are internally tangent to $O$, with their tangent points to $O$ in counterclockwise order. The small circles do not intersect each other. Among the two common external tangents of $O_1$ and $O_2$, let $l_{12}$ be the one which separates $O_1$ and $O_2$ from the other two circles, and let the intersections of $l_{12}$ and $O$ be $A_1$ and $B_2$, with $A_1$ denoting the point closer to $O_1$. Define $l_{23}, l_{34}, l_{41}$ and $A_2, A_3, A_4, B_3, B_4, B_1$ similarly. Suppose that the arcs $A_1B_1$, $A_2B_2$, and $A_3B_3$ have length $\pi$, $3\pi/2$, and $5\pi/2$ respectively. Find the arc length of $A_4B_4$.

10. Given a triangle $ABC$ with $BC = 5$, $AC = 7$, and $AB = 8$, find the side length of the largest equilateral triangle $PQR$ such that $A, B, C$ lie on $QR, RP, PQ$ respectively.