Time limit: 50 minutes.

Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.

No calculators.

1. If \( f(x) = (x - 1)^4(x - 2)^3(x - 3)^2 \), find \( f'''(1) + f''(2) + f'(3) \).

2. A trapezoid is inscribed in a semicircle of radius 2 such that one base of the trapezoid lies along the diameter of the semicircle. Find the largest possible area of the trapezoid.

3. A sector of a circle has angle \( \theta \). Find the value of \( \theta \), in radians, for which the ratio of the sector's area to the square of its perimeter (the arc along the circle and the two radial edges) is maximized. Express your answer as a number between 0 and \( 2\pi \).

4. Let \( f(x) = \frac{x^3 e^x}{1 - x^2} \). Find \( f^{(7)}(0) \), the 7th derivative of \( f \) evaluated at 0.

5. The real-valued infinitely differentiable function \( f(x) \) is such that \( f(0) = 1 \), \( f'(0) = 2 \), and \( f''(0) = 3 \). Furthermore, \( f \) has the property that
\[
\sum_{n=0}^{\infty} f^{(n)}(x) = 0
\]
for all \( n \geq 0 \), where \( f^{(n)}(x) \) denotes the \( n \)th derivative of \( f \). Find \( f(x) \).

6. Compute \( \int_{-\pi}^{\pi} \frac{x^2}{1 + \sin x + \sqrt{1 + \sin^2 x}} \, dx \).

7. For the curve \( \sin(x) + \sin(y) = 1 \) lying in the first quadrant, find the constant \( \alpha \) such that
\[
\lim_{x \to 0} x^\alpha \frac{d^2 y}{dx^2} = 0
\]
exists and is nonzero.

8. Compute \( \int_{\frac{1}{2}}^{2} \frac{\tan^{-1} x}{x^2 - x + 1} \, dx \).

9. Solve the integral equation
\[
f(x) = \int_0^x e^{x-y} f'(y) \, dy - (x^2 - x + 1)e^x.
\]

10. Compute the integral
\[
\int_0^\pi \ln(1 - 2a \cos x + a^2) \, dx
\]
for \( a > 1 \).