Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.
No calculators.

1. Let $a, b \in \mathbb{C}$ such that $a+b=a^2+b^2=\frac{2\sqrt{3}}{3}i$. Compute $|\operatorname{Re}(a)|$.

2. Consider the curves $x^2+y^2=1$ and $2x^2+2xy+y^2-2x-2y=0$. These curves intersect at two points, one of which is $(1,0)$. Find the other one.

3. If r, s, t, and u denote the roots of the polynomial $f(x)=x^4+3x^3+3x+2$, find
$$\frac{1}{r^2}+\frac{1}{s^2}+\frac{1}{t^2}+\frac{1}{u^2}.$$

4. Find the 2011th-smallest x, with $x>1$, that satisfies the following relation:
$$\sin(\ln x)+2\cos(3\ln x)\sin(2\ln x)=0.$$

5. Find the remainder when $(x+2)^{2011}-(x+1)^{2011}$ is divided by x^2+x+1.

6. There are 2011 positive numbers with both their sum and the sum of their reciprocals equal to 2012. Let x be one of these numbers. Find the maximum of $x+x^{-1}$.

7. Let $P(x)$ be a polynomial of degree 2011 such that $P(1)=0$, $P(2)=1$, $P(4)=2$, ... , and $P(2^{2011})=2011$. Compute the coefficient of the x^1 term in $P(x)$.

8. Find the maximum of
$$\frac{ab+bc+cd}{a^2+b^2+c^2+d^2}$$
for reals a, b, c, and d not all zero.

9. It is a well-known fact that the sum of the first n k-th powers can be represented as a polynomial in n. Let $P_k(n)$ be such a polynomial for integers k and n. For example,
$$\sum_{i=1}^{n} i^2=\frac{n(n+1)(2n+1)}{6},$$
so one has
$$P_2(x)=\frac{x(x+1)(2x+1)}{6}=\frac{1}{3}x^3+\frac{1}{2}x^2+\frac{1}{6}x.$$
Evaluate $P_7(-3)+P_6(-4)$.

10. How many polynomials P of degree 4 satisfy $P(x^2)=P(x)P(-x)$?