proco

May 12, 2019

Novice: Number of Solves

whatever	37	groot	13
thor	35	hawkeye	22
assemble	24	blackpanther	6
scarletwitch	26	thanos	4

Novice: Fastest Solves (minutes)

whatever	0	groot	35
thor	1	hawkeye	11
assemble	5	blackpanther	41
scarletwitch	8	thanos	62

Advanced: Number of Solves

assemble	29	blackwidow	5
ironman	24	asgardians	0
vision	9	hulk	0
captainamerica	4	spiderman	0
titan	5		

Advanced: Fastest Solves (minutes)

assemble	2	blackwidow	54
ironman	8	asgardians	N/A
vision	10	hulk	N/A
captainamerica	42	spiderman	N/A
titan	68		

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Speed Round

Solutions

Novice Solutions

thor

- We first look at how many cubes with side length “a” can fit along one direction of cubes with side length “b”
- This quantity is (b/a)--using integer division
- Thus we can fit (b/a) cubes in each direction, since a cube is symmetric
- We can fit a total of (b/a)*(b/a)*(b/a) number of cubes
- This solution runs in O(1) time

assemble

- Observations:
  - The most frequent substring will be of length 1
  - The least frequent substring will be the entire string (1 substring)
- For the least frequent substring, we just count the frequency of every letter and output the max
- For the most frequent substring, we just output the entire string
- This runs in O(n) time

scarletwitch

- Note that we can ignore trailing 1’s and leading 0’s. We can treat this as just chopping those digits off the string
- Otherwise we can just reverse the entire string and look at the string again.
- We don’t have to physically reverse the string, just look at it forwards or backwards
- Keep track of how many reverses we do
- This runs in O(n) time

groot

- Movement is deterministic and can be easily simulated in $O(n)$ time.
- Since there are n instructions and only 4 types of instructions, there are only 3n possible single-command changes, and we can just try all of them.
- This leads to an $O(n^2)$ solution.
- It is possible to update the end position in $O(1)$ time by keeping both the change in position and the change in orientation of each suffix and prefix of instructions. This was not necessary for the problem.

hawkeye

- Create a min heap/priority queue and insert all numbers into it.
- Repeat until min heap is empty:
  - extract two minimum elements from the heap
  - add their sum to the heap
  - Update the total cost
- We have n inserts and the loop is repeated n-1 times, each insert() and getMin() is O(log n). So the algorithm is O(n log n).
- We also accept $O(n^2 \log n)$ solutions that use an array/list and sort it after every insert().

blackpanther

- Notice the number of operations we can do is less than 9. We can brute force.
- Try all permutations of the order of numbers and see which ordering is maximal
- This $O(n! * n)$ solution runs in time since $n <= 9$

thanos

- Think about two planets with periods A and B (assume A<B). After D days, planet 1 has travelled D/A rotations and planet 2 has travelled D/B. If they're "in line" then their difference is an integer number of rotations:
- $D/A - D/B$ = an integer $k \rightarrow D(B-A)/AB = k \rightarrow D = kAB/(B-A)$
- We need all n planets in line, so planet 1 is in line with 2, 1 is in line with 3, … 1 is in line with n
- We get n-1 fractions

thanos

- Our final answer fraction must be an integer multiple of all these fractions. (A “Lowest Common Multiple” but for fractions)
- For two fractions: A/B and C/D → find <u>smallest</u> common denominator G, so A’/G and C’/G, then find the lcm of integers A’ and C’
- Use the Euclidean Algorithm to find GCF, then LCM(x,y)=xy/GCF(x,y)
- Find the lcm of the first two fractions. Then the lcm of this answer and the next fraction etc.
- This runs in O(n) time
  - Careful: numbers get large

Advanced Solutions

ironman

- The “relative ordering” of a sequence of numbers a_1, a_2,..., a_n is a permutation of 1,2,..., n if where the smallest number is replaced by 1, the second smallest is 2, ... , largest is n.
- Observation: If you swap two rows, the relative ordering of each row is unchanged. If you swap two columns, all of the relative orderings of rows change in the same way.
  - The same is true for the relative ordering of rows
- Output Yes if all the rows have the same relative ordering and all the columns have the same relative ordering (as other columns, can be distinct from the rows)
  - Note that any relative ordering is sortable. If there are two distinct relative orderings in rows, if you sort one, the other cannot be sorted.
- Runs in O(n^2log(n))

vision

- We can notice that the prime factorization of n^3 must be the same as the prime factorization c(m^2)
- We can greedily choose the smallest numbers to build n and m
- Sieve of Eratosthenes to find primes from 1 to 10^6
- Prime factorize c
- To build n and m consider each p^e in X:
    - If e=1,2: p^e should be in n and m
    - If e > 2: the smallest exponents for p in n and m are:
        - If p = 3k: n has p^k, m has p^0
        - p= 3k+1: n has p^{k+1}, m has p^1
        - p = 3k+2: n has p^{k+2}, m has p^2
- Careful: This process might get n=m. If so, multiply n by 4 and m by 8
- O(Nlog(N)+Q)

captainamerica

- Type 0 operations add exactly one triangle when (u, v) is an existing edge.
- Type 1 operations will only remove triangles.
- So, there can only be up to q triangles, and each triangle is inserted and removed at most once.
- For every edge (u,v) store the set of vertices that are connected to u and v, with a set (i.e. all the triangles including (u,v))
- Process all the operations in O(q log q) time.

titan

- Can assume all heights will be the same (as the max height)
- For two towers at (x_1,y_1,z_1) and (x_2,y_2,z_2) find an equation for the minimum height they need to be to see each other:
  - Angle between them = $\arccos(x_1x_2+y_1y_2+z_1z_2)$
  - Min Height = 1/cos(angle/2) - 1
- Solution 1: Kruskal's Algorithm
  - Sort all pairs of towers by increasing tower height needed
  - Use a union-find data structure to find the minimum height needed to make everything one connected component
- Solution 2: Binary Search
  - For a given height H, link all the pairs with min height <= H
  - See if everything is connected
- Runs in O(N^2 logN) and O(N^2 log(max height)) respectively

blackwidow

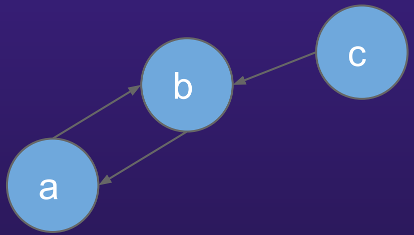
- Maintain a map mp from box number to expected number of balls
- Initially the expected number of balls for the ith box is a_i
- To update mp after operation [l, r], iterate over the keys in that range and compute the sum of the values
- After iterating over a key, remove it from the map
- Set mp[l] and mp[r] to sum/2 (if l = r we don’t do anything)
- Runtime is O((n + m)logn)
    - For each operation, we add at most 2 entries to the map so the total number of additions to the map is at most n + 2*m
    - We can only remove an entry from a map once, so the total number of removals across all operations is O(n + m)

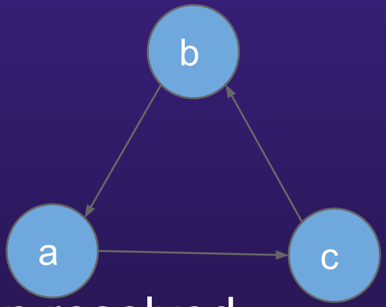
asgardians

- Create graph of 26 nodes (one for each letter)
- If we have to change letter c_1 to c_2 at some point, create directed edge from c_1 to c_2.
- Impossible cases:
  - Some letter needs to be changed to two different letters
    - S = ab**b**a and T = ab**c**a
  - Edges (of all 26 nodes) form a set of simple cycles (self-loops count as cycles) unless everything is a self-loop (in which case the strings are identical)

asgardians

- Answer: number of edges in the graph (excluding self-loops) + number of simple cycles (excluding self-loops)
- Resolve tree components: process changes from root outwards
- Resolve flower components: change *a* to *c*, then resolve the tree rooted at *a*





- Resolve cycle components:
  - Let *d* be *a* leaf node of some flower that has already been resolved
  - Change *a* to *d* and resolve the tree rooted at *a*
  - Change *d* back to *a* (creates an additional unit of cost for a cycle)
- This solution runs in O(n)

hulk

- Only need to consider grids where all diagonals are the same character

```
abcd        abcd
qwer   ->   bcdr
asdf        cdrf
xzcv        drfv
```

- Hence, all we need is a sequence of $2n - 1$ characters that does not contain any bigram or trigram as a substring.
- If $n = 1$, any character works.
- Otherwise, build a graph!

hulk

- Make a node *ab* if the bigram *ab* is not in the list of bigrams.
- Make an edge from *ab* to *bc* if the trigram *abc* is not in the list.
- Find a path containing 2n - 2 nodes in this graph.
- If there is a cycle, you can make a path of any length by repeating it.
- Otherwise, the graph is a directed acyclic graph, so you can find the longest path in it in O(V + E) time, where V is at most 26^2 and E is at most 26^3.

spiderman

- For a fixed partitioning scheme, always pick *x* to be the most frequent value. So cost = length of array minus sum of largest frequencies.
- Define dp(i, j) = sum of frequencies for each subarray of the first *i* elements into *j* subarrays AND a_i is the value we are using for the *j*th subarray
  - Can use the second condition because there always exists a solution with the right endpoint of every subarray equal to the most frequent in that subarray

spiderman

- At a_i, we have two cases:
  - Start a new subarray for the *i*th element:
    - dp(i, j) = 1 + max(dp(i', j - 1)) for all $i' < i$.
  - Extend some subarray:
    - dp(i, j) = 1 + dp(prv[a_i], j) where prv[x] is the most recent index in which we saw a_i
- Answer = N - max(dp(i, j)) for all *i* and *j*
- This runs in O(NK)