ABSTRACT:
In a contest to describe the largest integer in the space of a single sheet of paper, a clever contestant may write “one greater than the maximum of the others,” but we run into inconsistencies if multiple contestants use this same strategy. When rules are introduced to prevent inconsistencies, strategies for defining large numbers typically use functions with high growth rates, such as iterations of the exponential function. We expand on this idea to get a hierarchy of rapidly-growing functions where the next function is obtained by iterating the previous one, and then taking limits. In this way, infinite ordinal numbers are used to define increasingly fast-growing computable functions on the positive integers, providing a powerful tool for defining numbers so large they stretch the imagination. These functions are more than just a curiosity; they are used in mathematical logic to measure the complexity of proofs, displaying a strong connection between provability and computability. This leads to a strategy of unraveling mathematical axiom systems to define large integers, and gives us a way to take the idea from the "one greater than the maximum of the others" strategy and apply it in a consistent context. Through this exploration of strategies for defining large integers, this talk will give an overview of major results in logic and set theory; no prior knowledge of these fields is assumed.