How to Count the Rationals
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ABSTRACT:
In the 1800s, Georg Cantor proved that the rational numbers are “countable”: they can be listed in such a way that each rational number occurs exactly once in the (infinite) list. The usual proof represents the rational p/q by the point (p,q) in the upper half-plane, then zig-zags through all the points, skipping those where p and q have a common factor, and listing the remaining points in the order visited. Questions like the following are difficult to answer:

1. What is the rational number immediately after a given number?
2. What is the 125th rational number in the list?
3. Where does 5/17 occur in the list?

We will describe a different way to count the rational numbers. The alternative will give a bijection for which the above questions are easy to answer.