Partial Differential Equations in Fluid Dynamics
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ABSTRACT:

The equations describing the velocity field of an incompressible fluid have been known since around 1820, and pose many challenges to mathematicians. For instance, we can prove that solutions to these equations exist for a short time interval; however, we can NOT currently prove that these solutions do not develop singularities (or ``shocks'') over long periods of time. This is the $1,000,000 Clay problem. Loosely speaking, if you stir a cup of water, we cannot theoretically rule out the possibility that it will explode after long time!

Another problem is energy conservation. One would expect that an ```ideal`` fluid conserves energy. Physicists observed (and expect) that a ```turbulent enough`` ideal fluid will still dissipate energy! Mathematically, this comes from the fact that ```integration by parts`` need not hold when the integrands are not smooth enough, and dates back to a (not completely settled) conjecture by Onsager in 1949.

I'll give a non-technical talk introducing some interesting problems, and briefly motivate some of the issues involved.

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