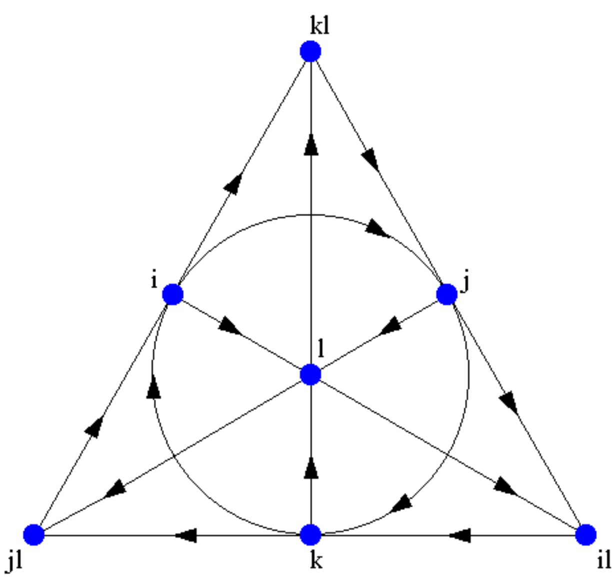
The SUMO Speaker Series for Undergraduates

(food from Pizza Chicago)
Wednesday, February 25th
5:15-6:15, room 380C

Real Numbers, Complex Numbers, then what?

Professor Soren Galatius



ABSTRACT:

Given two points in the plane, how do you multiply them? Thinking of the plane as the complex numbers gives one answer: The product of $P = (a,b)$ and $Q = (c,d)$ is $(ac - bd, ad + bc)$. Multiplying points in this way has all sorts of nice and desirable properties, these are the properties that make complex numbers so useful.

What about points in 3-space? Is there a nice way to multiply two points $P = (a,b,c)$ and $Q=(d,e,f)$ and get a third? Perhaps this could give us a new number system beyond the real complex numbers, that would be even more useful? It turns out the answer is NO! There is not a way of multiplying points in 3-space which is even remotely as nice as the one in 2 dimensions. What about 4 dimensions then? It turns out the answer is YES. It also turns out the answer is YES in 8 dimensions, and that it is NO in all other dimensions.

I will explain more precisely what I mean by a "nice" multiplication, and how to get one in dimension 4 and 8. You might wonder how one answers this question in the cases where the answer is "NO" --- how does one prove that a nice multiplication does *not* exist? Surprisingly, the answer will almost immediately take us out of the world of algebra, and far into the world of "geometry and topology".

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