Equidistribution of arithmetic points on the unit sphere

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ABSTRACT:
Gauss proved that any positive integer $d$ not of the form $4^k(8l-1)$ is a sum of three squares: the equation $d = a^2 + b^2 + c^2$ has a solution $(a, b, c)$ in integers. A natural question concerns the distribution of these solutions as $d$ grows. More precisely, for any such $d$ one scales the solutions down to the unit sphere, and so gets the set of unit vectors

$$V(d) = \left\{ \frac{1}{\sqrt{d}} (a, b, c); a^2 + b^2 + c^2 = d \right\}.$$ 

Each $V(d)$ is a finite set on the unit sphere, and the remarkable fact is that when $d$ goes to infinity these points are well-distributed on the sphere, and in particular visit infinitely often the neighborhood of any point of the sphere. We will introduce the notion of equidistribution to make this precise, and sketch the connection of this problem with modular forms, which are useful analytical objects to attack many related arithmetic problems, and whose properties are both (very) algebraic and analytic in nature.