1. **Answer:** \( \frac{1+\sqrt{5}}{2} \)

Let \( x = \sqrt{1 + \sqrt{1 + \sqrt{1 + \ldots}}}. \) Then \( x^2 = 1 + \sqrt{1 + \sqrt{1 + \ldots}}. \) Thus \( x^2 = x + 1. \) The positive root of \( x^2 - x - 1 = 0 \) is \( \frac{1+\sqrt{5}}{2}. \)

2. **Answer:** \( \frac{334703}{1665000} \)

This is simply \( \frac{2010}{1000} + \frac{1}{1000} \cdot \frac{228}{999} = \frac{334703}{1665000}. \)

3. **Answer:** 19801 and 20201

Notice that \( 4x^4 + 1 = 4x^4 + 4x^2 + 1 - (2x)^2 = (2x^2 + 2x + 1)(2x^2 - 2x + 1). \) Setting \( x = 100, \) we have that \( 40000001 = 19801 \cdot 20201. \)

4. **Answer:** \( \pm 123 \)

Note that \( (x + \frac{1}{x})^2 = x^2 + \frac{1}{x^2} + 2 = 9. \) Thus, \( x + \frac{1}{x} = \pm 3. \) Therefore,
\[
x^5 + \frac{1}{x^5} = \left(x + \frac{1}{x}\right)^5 - 5 \left(x^3 + \frac{1}{x^3}\right) - 10 \left(x + \frac{1}{x}\right)
= \left(x + \frac{1}{x}\right)^5 - 5 \left(x + \frac{1}{x}\right)^3 + 5 \left(x + \frac{1}{x}\right)
= \pm 123
\]

5. **Answer:** 17

The open lockers will be the ones with an odd number of odd divisors. These numbers are of the form \( 2^k \cdot n^2, \) where \( n \) is odd. We can simply check that the open lockers are numbered
\[
1, 2, 4, 8, 9, 16, 18, 25, 32, 36, 49, 50, 64, 72, 81, 98, 100.
\]

6. **Answer:** 1027

If \( S(n) \) is the \( n \)th partial sum, note that if \( m \) is the \( k \)th triangular number, \( S(m) = k^2. \) Since \( 44^2 = 1936 \) and \( 45^2 = 2025, \) we want to begin our search at \( 44(44 + 1)/2 = 990. \) Because \( (2010 - 1936)/2 = 37, \) 37 more 2s are needed, so the needed term is \( n = 990 + 37 = 1027. \)

7. **Answer:** 21, 26, 31, 36, 41, 46

\[
6x + 5 \equiv -19 \mod 10 \\
6x \equiv -24 \mod 10 \\
x \equiv -4 \mod \frac{10}{\gcd(10, 6)} \\
x \equiv -4 \mod 5 \\
x \equiv 1 \mod 5
\]

That is, \( x \) is in the form \( 5k + 1 \) where \( k \) is an integer.

8. **Answer:** \( 3^{n+1} - 2^{n+1} \)

We use the fact that if \( P(x) \) is a polynomial of degree \( n, \) then \( P(x+1) - P(x) \) is a polynomial of degree \( n - 1. \) Define \( \Delta P(x) = P(x+1) - P(x). \) By induction on \( m, \) it can be easily proved that \( \Delta^m P(x) \) is a polynomial of degree \( n - m \) such that \( \Delta^m P(k) = 2^m \cdot 3^k \) for \( 0 \leq k \leq n - m \) when \( 0 \leq m \leq n. \) Moreover, \( \Delta^{n+1} P \) is identically zero, since \( \Delta^n P \) is degree zero and applying \( \Delta \) to constants leaves zero. Thus
\[ P(n+1) = P(n) + (P(n+1) - P(n)) \]
\[ = P(n) + \Delta P(n) \]
\[ = P(n) + \Delta P(n-1) + (\Delta P(n) - \Delta P(n-1)) \]
\[ = P(n) + \Delta P(n-1) + \Delta^2 P(n-1) \]
\[ = P(n) + \Delta P(n-1) + \Delta^2 P(n-2) + (\Delta^2 P(n-1) - \Delta^2 P(n-2)) \]
\[ = P(n) + \Delta P(n-1) + \Delta^2 P(n-2) + \Delta^3 P(n-2) \]
\[ = \ldots \]
\[ = \sum_{i=0}^{n} \Delta^i P(n-i) + \Delta^{n+1} P(0) \]
\[ = \sum_{i=0}^{n} 2^i 3^{n-i} \]
\[ = 3^{n+1} - 2^{n+1}. \]

9. **Answer:** 31

Factor the equation as \((x + 2)(y - 5) + 10 = 30\), or \((x + 2)(y - 5) = 20\). \(x\) must be 2 less than a factor of 20. The solutions for \(x\) are thus 2, 3, 8, and 18, which sum to 31.

10. **Answer:** \(\frac{2}{1005}\)

We can rewrite this equation as

\[
\frac{x^2}{x^2 - 1} + \frac{x^2}{x^2 - 2} + \frac{x^2}{x^2 - 3} + \frac{x^2}{x^2 - 4} = \]
\[= \frac{1 + (x^2 - 1)}{x^2 - 1} + \frac{2 + (x^2 - 2)}{x^2 - 2} + \frac{3 + (x^2 - 3)}{x^2 - 3} + \frac{4 + (x^2 - 4)}{x^2 - 4} \]
\[= (2010x - 4) + 4 = 2010x. \]

We divide by \(x\); this makes us lose the solution \(x = 0\), but this does not affect the sum of solutions. Therefore, we have

\[
\frac{x}{x^2 - 1} + \frac{x}{x^2 - 2} + \frac{x}{x^2 - 3} + \frac{x}{x^2 - 4} = 2010 \]

Clearing denominators yields the polynomial equation

\[
 x((x^2 - 2)(x^2 - 3)(x^2 - 4) + (x^2 - 1)(x^2 - 3)(x^2 - 4) +
 (x^2 - 1)(x^2 - 2)(x^2 - 4) + (x^2 - 1)(x^2 - 2)(x^2 - 3)) \]
\[= 2010(x^2 - 1)(x^2 - 2)(x^2 - 3)(x^2 - 4) \]

The solutions that we want are therefore the roots of the polynomial

\[2010x^8 - 4x^7 + \text{(lower order terms)} = 0\]

By Vieta’s formulas, the sum of the roots of this polynomial equation is therefore \(\frac{4}{2010}\).