

1. In ancient Roman games, gladiators were pitted against velociraptors in turn-based combat. In a standard match, one velociraptor fought n gladiators. The raptor attacked first, and hit one gladiator per turn, killing him instantly. One gladiator attacked per turn, hitting the speedy raptor with probability $\frac{1}{2}$. If it took ten hits to kill the raptor, what is the smallest number of gladiators the emperor could have sent into the arena in order to have at least $\frac{1}{2}$ probability of killing the velociraptor?
2. In ancient Cartesia, city blocks were perfect squares, all with the same side length. East-west streets were named First, Second, Third, and so on, while north-south streets were named Abel, Bernoulli, Cauchy, and so on. Carrier pterodactyls were used to deliver large packages, since they could always fly the shortest distance between two points. If a pterodactyl picked up two packages at First and Abel, then dropped the first out of the sky onto Fourth and Erdős as it turned toward its next destination at Eighth, by what angle did it turn above the first drop-off?
3. Two numbers are chosen randomly from the interval $[0, 1]$. What is the probability that they differ by more than their average?

4. Given that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$, evaluate $\sum_{n=1}^{\infty} \frac{1}{n^2+8n+16}$.

5. Three X's, three Y's, and three Z's are placed randomly in a 3×3 grid. What is the probability that no row or column will contain two of the same letter?
6. A function f maps every sequence of integers to another sequence of integers as follows:

$$(f(a))_n = \begin{cases} 1 & \text{if } n = 0 \\ na_{n-1} & \text{if } n \neq 0 \end{cases}$$

If the sequence b_n is a fixed point of f , what is $b(2008)$?

7. Lord Voldemort only does two things all day: curse Muggles, and kick puppies. Each Muggle he curses has a 50% chance of dying while a puppy kick is always successful. Each dead Muggle gives him 3 units of satisfaction and each kicked puppy gives him 2 units. If an even number of Muggles die, he doubles his satisfaction from each of them. If he can curse one Muggle or kick one puppy per hour, how many Muggles should he curse in a day to maximize his expected satisfaction?
8. Bill has an infinite amount of time on his hands, which he spends by drawing fractals in full detail. Beginning with a unit square, he inscribes a circle in every square he draws and inscribes a square in every circle he draws. What is the total length he draws?
9. Two complex numbers z_1, z_2 have purely imaginary product and purely real quotient. How many ordered pairs (z_1, z_2) are there such that $|z_1| = |z_2| = 1$?
10. Evaluate $S = \sum_{x=0}^{\infty} \sum_{y=0}^{\infty} \frac{1}{z^{x+y+|x-y|}}$.